Spatial Wage Differentials, Geographic Frictions and the Organization of Labor within Firms

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Abstract

This paper studies the spatial organization of firms, both theoretically and empirically. Two new facts in Danish register data motivate the analysis: (i) firms have become increasingly spatially fragmented, and (ii) headquarters (HQ) establishments have become more manager-intensive. We develop and estimate a structural model in which firms allocate labor across establishments and produce non-rival, manager-intensive HQ services. Identification relies on exogenous variation in labor supply induced by commuting-augmented immigration shocks. We estimate elasticities of substitution across establishments of -9.8 for workers and -1.1 for managers, consistent with firms reallocating general labor more easily than managerial inputs. Our decomposition shows that rising managerial wages at HQs-interacted with firm-level scale effects-explain about half of the observed increase in HQ managerial intensity, highlighting the importance of intangible internal inputs in shaping firm spatial structure.

JEL: D22, J23, L22, L23, R30.

Keywords: firm organization, multi-establishment firms, wages, communication costs, agglomeration.

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1 Introduction

Firms are becoming increasingly spatially fragmented, reshaping labor markets and the organization of production. This fragmentation has contributed to a growing share of economic activity being generated by multi-establishment (ME) firms, especially in developed economies (Kim, 1999; Charnoz et al., 2018; Acosta and Lyngemark, 2021). Today, ME firms account for half of all private-sector employment in Denmark and 60% of private employment in France (Charnoz et al., 2018). These firms typically concentrate their headquarters (HQs) in dense urban areas (Strauss-Kahn and Vives, 2009) and propagate local economic shocks across space (Giroud and Mueller, 2019; Gumpert et al., 2022). Consequently, their organization may explain key empirical patterns, such as the sorting of high-skill workers into high-wage cities and other spatial phenomena (Acosta and Lyngemark, 2021; Spanos, 2019). These trends also raise fundamental questions about the forces shaping firm structure, the allocation of labor across locations, and the role of spatial wage disparities in these dynamics.

This paper studies how regional wage disparities and communication costs affect the internal organization of firms, both theoretically and empirically. This study is motivated by two key changes in the internal spatial organization of firms. First, firms have become more spatially fragmented. In Denmark, the number of establishments per firm has increased by 21% since 1981, and the average distance between HQs and satellite establishments has doubled (Acosta and Lyngemark, 2021).¹ Second, HQs have become increasingly manager-intensive relative to other establishments, even as managerial wages in these locations have risen. Between 1996 and 2016, the ratio of managers to workers at firms' HQ rose by 13–16% relative to satellite establishments.

To explain these patterns, we develop a structural model in which firms allocate labor across establishments while considering regional wage disparities, HQ-produced services, and communication costs. We find that, in response to a wage shock, ME firms primarily adjust by reallocating non-managerial workers across establishments, whereas single-establishment (SE) firms can only react by changing their worker composition within their unique location. Moreover, we find that the observed increase in managerial intensity at HQ is largely driven by intangible inputs–HQ services and managerial knowledge–alongside rising spatial wage differentials.

This paper makes three main contributions. First, it provides new empirical evidence on firm fragmentation and managerial specialization using Danish matched employer-employee data. Second, using a novel commuting-augmented immigration share shifter, it estimates the elasticity of substitution across occupations and locations, showing that managers and HQ services are more location-sensitive than general labor. Third, it develops and estimates one of the first structural models in the literature to examine how spatial frictions and wage shocks influence the behavior of ME firms and demonstrates how firm-level scale economies drive intrafirm labor reallocation. More broadly, our findings contribute to the understanding of how ME firms shape a country's economic geography.

¹Similar trends have been observed in other high-income countries, including the United States (Kim, 1999; Davis and Henderson, 2008) and France (Charnoz et al., 2018).

To understand the observed changes in the internal spatial organization of firms, we develop a model of firm location and labor demand. Building on the literature on multinational firms (Helpman, 1984; Horstmann and Markusen, 1987; Markusen, 2002), the model allows firms to determine the number, location, and labor composition of their establishments. A central feature of our theory is the role of HQ services—intangible, manager-intensive inputs that facilitate firm-wide coordination but attenuate with geographic distance. Since these services are non-rival, they generate within-firm economies of scale that encourage the expansion of ME firms. Recent research highlights the importance of internal transfers of such intangible inputs in explaining firm boundaries and productivity increases (Atalay et al., 2014). In our model, spatial wage differentials create incentives to open new establishments and reallocate workers across locations. However, expanding into new locations comes with additional production and communication costs, which constrain firm growth.

We use this model to examine how changes in wages across locations and communication costs shape the allocation of workers across establishments. Our model predicts that when HQ locations experience a positive wage shock, ME firms respond by reallocating non-managerial workers to cheaper satellite establishments. As these establishments expand, they require more HQ services. This rising demand for these non-rival inputs, in turn, increases the relative demand for managers at the HQ. When the elasticity of substitution for managers across locations is low and HQ services are critical to production, this scale effect can outweigh the direct negative price effect. As a result, higher managerial wages at HQ can paradoxically lead to more manager-intensive HQs. Consistent with this mechanism, the model also predicts that the elasticity of substitution across locations is greater for workers than for managers and that substitution for managers declines with distance from the HQ.

We take this model to the data using matched employer-employee register data for Denmark from 1986 to 2016. This dataset allows us to track firm establishments, their locations, and their workforce composition. To test the model's main predictions, we begin by estimating reduced-form relative labor demand equations within and across locations. For econometric identification, we employ an augmented version of the widely used immigration share shifter instrument (Foged and Peri, 2016; Lewis, 2011; Baum-Snow et al., 2018). Since we seek labor supply shocks at establishment locations, we map immigration shocks from municipalities of residence to workplace municipalities using commuting flows. This instrument provides a source of exogenous variation in the supply of managers and workers across and within municipalities. To the best of our knowledge, this commuting-augmented immigration instrument represents a novel identification strategy in this literature.

Our empirical analysis reveals several key findings. First, we find that SE firms exhibit an elasticity of substitution between managers and workers of -1.1, whereas for establishments in ME firms, the estimate is not significantly different from zero. This result aligns with the intuition that ME firms can adjust to labor market shocks by reallocating workers both within and across establishments, while SE firms can only adjust labor composition within a single location. Second, we find evidence that as HQ locations become more expensive, they become relatively more manager-intensive. Third, our results suggest that firms respond more to wage differentials for workers across locations (elasticity of -9.8) than to differences in manager wages (elasticity of -1.1). Furthermore, the spatial elasticity of substitution for managers declines as the distance from HQ increases, reinforcing the idea that managerial coordination is more location-sensitive than general labor substitution.

From the model, we also derive a system of relative labor demand equations that allows us to estimate its structural parameters. The estimates suggest strong evidence of decreasing returns to scale final goods production and rising communication costs since 1996. This increase in communication costs may be driven by growing complexity in internal firm information flows (Brynjolfsson and McAfee, 2014) and the broader spatial fragmentation of firms (Acosta and Lyngemark, 2021).

The structural estimation enables us to quantify the relative contributions of wage differentials and communication costs to the observed rise in managerial intensity at HQ. Our results suggest that the standard labor demand channel, i.e., changes in the wages of managers relative to workers, explains only 15.4% of the observed change. This small share indicates that treating firms as single-location entities, as standard models do, overlooks crucial spatial dynamics. By failing to account for how firms reallocate labor across establishments, these models may miss key mechanisms driving aggregate spatial patterns. In contrast, changes in communication costs explain approximately 51%, while rising managerial wages at HQ relative to non-HQ locations account for 43%. These findings highlight the role of firm-level scale economies in driving intra-firm labor reallocation, particularly when interacting with rising relative wages for HQ-based managers. To test whether increasing HQ managerial productivity advantages could be explaining these trends, we estimate an alternative specification of the model incorporating this channel. Our results remain robust to this modification.

Literature Review Our paper relates to five broad strands of literature. First, to the literature on firm organization (Radner, 1992; Becker and Murphy, 1992; Garicano, 2000; Bresnahan et al., 2002; Bloom et al., 2014). This research explores the relationship between communication costs, firm structure, and knowledge hierarchies, emphasizing the role of managers in processing information and solving problems. Our paper also aligns with recent empirical work on internal input markets in ME and vertically integrated firms (Giroud, 2013; Atalay et al., 2014; Tate and Yang, 2015; Charnoz et al., 2018; Cestone et al., 2022; Gumpert et al., 2022). This literature primarily investigates how reductions in communication costs impact establishment-level outcomes, with a strong focus on econometric identification of theoretical predictions.

Closely related to us, Charnoz et al. (2018) and Gumpert et al. (2022) analyze the effects of geographic frictions on firm organization in France and Germany, respectively, using the introduction of high-speed train routes as a quasi-experimental setting. Similarly, Jiang (2022) finds that improved internet access in the U.S. facilitated the creation of new establishments by manufacturing firms. Our contribution to this literature lies mainly in the introduction of spatial wage disparities as a novel mechanism influencing the internal spatial organization of firms.

Second, within urban economics, this paper builds on research examining the location de-

cisions of multi-unit firms. On the theoretical side, studies such as Ota and Fujita (1993), Duranton and Puga (2005), Rossi-Hansberg et al. (2009), and Gokan et al. (2019) analyze how reductions in communication costs drive firm fragmentation and reshape the spatial distribution of activities within and across cities. On the empirical side, Henderson and Ono (2008), Davis and Henderson (2008), Bartelme and Ziv (2024), and Oberfield et al. (2024) investigate the determinants of HQ location and firm agglomeration. These two strands of literature have largely developed independently, partly due to the lack of suitable data for estimating spatial firm models. This paper bridges the gap by developing a firm-level model that can be estimated using detailed establishment-level data, enabling an evaluation of different mechanisms in shaping firm organization.

Third, our paper contributes to the literature on spatial sorting. This literature documents the increasing concentration of high-skilled workers and high-productivity firms in high-wage geographic areas (Moretti, 2004; Combes et al., 2008, 2012; Diamond, 2016; De La Roca and Puga, 2017; Baum-Snow et al., 2018; Gaubert, 2018; Eckert, 2019). We extend this literature by showing that similar sorting patterns occur within firms: HQs have become more managerintensive despite rising managerial wages in those locations. Since HQs are disproportionately located in denser labor markets with faster-growing managerial wages, this increasing withinfirm polarization may reinforce broader spatial polarization across labor markets (Acosta and Lyngemark, 2021).

Fourth, our paper relates to advances in the international trade literature on multinational enterprises. This literature primarily examines firms' decisions between exporting, outsourcing, and offshoring, emphasizing differences in market access, production costs, and transportation costs as key determinants (Helpman, 1984; Markusen, 2002; Antràs et al., 2006; Antràs and Yeaple, 2014; Fort, 2017). We view the rise of multinational enterprises as an extreme case of firm fragmentation. In particular, our work connects to the theoretical framework of Antras et al. (2006), who model team formation in an offshoring context.

Finally, this paper relates to a broad management science literature on the organization of modern corporations and the role of HQs. This literature typically focuses on two types of firms: multibusiness and multinational (Menz et al., 2015). The multibusiness perspective examines firms operating across multiple product markets within national borders (Chandler, 1969; Fligstein, 1985; Chandler, 1991; Collis et al., 2007), while the multinational perspective studies firms expanding across geographic markets (Laamanen et al., 2012; Collis et al., 2012).

The rest of the paper proceeds as follows. In Section 2, we present our data and describe our sample. In Section 3, we show the increase in managerial intensity at firms' HQs, relative to non-HQ establishments. In Section 4, we develop our theoretical model and discuss its main predictions. Section 5 presents the empirical and identification strategies, while Section 6 presents our reduced form and structural results. Section 7 concludes.

2 Data Description

Our data contain the full population of employers and employees collected by Statistics Denmark (DST) between 1981 and 2016. We give a more detailed description of these data in Acosta and Lyngemark (2021), but describe their main elements here. We use establishment records from the Integrated Database for Labor Market Research (IDA), which contains all matches between employees and their job (establishment and firm) with the most hours in the last week of November. We match these data with the General Firm Statistics (FIRM) and the Firm Accounting Statistics (FIRE). Since the data have unique firm and establishment identifiers, we can determine whether a firm has one or multiple establishments.

We observe the municipality and traffic zone of each establishment, as well as the municipality reported by the firm in its accounting records.² We calculate the distance between two establishments as the distance between the centroids of their respective traffic zones. Since DST does not report which establishment is the firm's headquarters (HQ), we classify an establishment as the HQ if its municipality matches the one listed in the firm's accounting records and it employs at least five workers.³

The IDA also includes information on each worker's hourly wage, tenure, and labor market experience. Wages are deflated using the 2015 national Consumer Price Index. For each individual, we keep only their main job, as defined by DST as the job in which the person normally works the most hours. If two jobs have the same number of hours, the one with the higher wage is selected as the main job. We link our data to registers containing information on age, immigration status, and educational attainment, using the Population, Immigration, and Education Statistics registers, respectively.⁴ We restrict the sample to individuals between 15 and 80 years old. Our data also allow us to compute commuting flows between municipalities by year and occupation. As will be discussed in Section 5, these commuting flows, combined with immigration shocks, form the basis of our identification strategy.

To distinguish between managers and non-managers, we use two variables: (i) PSTILL/P-SOC from IDA, which defines the primary job for each worker in terms of their position, and (ii) DISCO88 from the Labor Classification Module (AKM).⁵ We categorize an individual as a

 $^{^{2}}$ In 2007, Denmark implemented a major structural reform that reduced the number of municipalities from 270 to 98. While most municipalities were merged, approximately 10 were split in different ways, depending on the outcomes of local parish elections. See Indenrigs- og Sundhedsministeriet (2004) and Vrangbæk (2010) for further details on the 2007 Danish Structural Reform. To ensure consistency in geographic units over time, we construct a crosswalk linking pre- and post-reform municipalities and apply it to all years prior to 2007. This crosswalk is available upon request. Following Foged and Peri (2016), we treat Copenhagen and Frederiksberg as a single municipality, given that the latter is a geographic enclave within the former. Traffic zones are 907 geographic units smaller than municipalities, defined by the Technical University of Denmark's National Transport Model.

 $^{^{3}}$ Using this definition, we are able to identify a headquarters for 96% of firms in our sample. The remaining 4% are typically firms with multiple establishments in the reported municipality. In such cases, we assign HQ status to the establishment with the highest concentration of: (i) managers, (ii) high-wage earners, (iii) workers holding a master's or doctoral degree, and (iv) workers with a technical or bachelor's degree.

 $^{^{4}}$ For population statistics, we use the FAIN register up to 2006 and the BEF register from 2007. These registers also contain variables of immigration date, which we complement with the IEPE register. For education statistics, we use the UDDA register.

⁵The DISCO classification changes in the registers between 2009 and 2010 from DISCO88 to DISCO08. These changes affected mostly the three- and four-digit-level codes, while we use mostly two-digit occupations. More

manager if she is categorized as a manager or top executive by either classification.

Taken together, these registers allow us to: (i) track each establishment over its lifetime, (ii) link establishments to firms and determine whether they are part of a multi-establishment firm, (iii) observe their location, and (iv) characterize its workforce by occupation and other worker attributes. We restrict our analysis to firms in manufacturing, transportation, business services, and finance, insurance, and real estate (FIRE), which together account for approximately 53% of national employment in Denmark over the past four decades. Our sampling criteria follow those used in Acosta and Lyngemark (2021).⁶ Since our identification strategy uses data between 1986 and 1994, we use data from 1996 onward for the descriptive statistics and estimations.

To obtain consistent measures of establishment-occupation-year level wages, we must restrict our sample further. We keep only workers with hourly wages between 50 and 5,500 DKK⁷ and drop those whose reported wage has strong measurement error, as indicated by the data.⁸ Then, we drop firms with less than 3 workers, or that had 1 to 3 workers for at least 66% of our sample. We drop these firms to avoid self-employees, small family businesses, shell companies and empty establishments.⁹ Finally, since our model assumes that all establishments have a positive number of workers and managers, we drop establishments for which we could not identify a manager. These correspond to around one third of establishments and hold around one sixth of the workforce from our sample. In our final dataset, we have on average 10,802 firms and 12,865 establishments each year, which contain around 562,501 workers. In total for the entire period 1996 to 2016, we have approximately 227k firm-year, 270k establishment-year, and almost 12 million worker-year observations.

Table A1 presents descriptive statistics for our sample. In Section B from the Online Appendix, we briefly discuss the spatial distribution of workers and HQ in Denmark and present a map of its commuting areas. The commuting areas of the four main municipalities account for 69% of HQ of ME establishments, and 51% of workers.¹⁰ This concentration of HQ establishments is not unique to Denmark. Strauss-Kahn and Vives (2009) show that HQ establishments in the United States are disproportionally located in denser and the largest metropolitan areas.

Since we classify all workers as either managers or non-managers, regardless of other individual characteristics, there is substantial heterogeneity within each group. This is particularly pronounced among non-managers, a category that includes a wide range of occupations such as

information on the crosswalk used is available on request.

⁶We dropped (i) establishments with no reported location, (ii) those located in sparsely populated islands, (iii) establishments with 1 or 2 employees or that only appear one year, and (iv) firms with more than 99 establishments or that exhibit large jumps in the total number of establishments across years (mostly outliers in the data from 1985 to 1988). Establishment do not report location when their workers do not have a fixed workplace (e.g., delivery or cleaning services). Table A5 in Acosta and Lyngemark (2021) provide more details.

⁷This corresponds to hourly wages between 7 and 770 in 2015 U.S. Dollars, restricting the bottom 0.1% and the top 0.01% tails of the distribution.

⁸Until 2008, the IDA register contained a variable (TLONKVAL) measuring the uncertainty of hourly wages. We drop observations with a value of zero ("zero hours worked") or higher than 100 ("useless quality").

⁹Since these atypical firms and establishments cannot be cleanly identified in the data, we exclude them to avoid including entities that do not operate as a standard model of the firm would predict. For example, see Bertrand et al. (2008)'s study on family firms.

¹⁰Commuting areas are defined by Nielsen (2005) based on commuting flows across municipalities from 2004 and have on average 4 municipalities. We present them in Figure A5 in the Appendix B.

secretaries, laborers, and engineers.

To improve comparability of workforces across establishments, we estimate a quality-adjusted average wage for each establishment-year by regressing the logarithm of the raw hourly wage of a worker p of occupation c (manager or non-manager, separately) in establishment i located in j at time t (log ω_{pcjkt}) on observable characteristics and establishment-year fixed effects, as follows:

$$\log \omega_{pcijt} = X_{pct}\beta^c + \log w_{cijt} + \varepsilon_{pcijt},\tag{1}$$

where the vector X_{pct} includes both time-variant and invariant worker characteristics including gender, region of origin, education interacted with a polynomial of degree two of experience and tenure, detailed occupation and job position dummies; log w_{cijt} denotes an establishment-year fixed effect for workers of occupation c. We use the (exponential of the) predicted values of these fixed effects \hat{w}_{cijt} as the measure of establishment-level quality-adjusted wages in all of our empirical exercises. Finally, ε_{pcijt} represents the error term. Besides from the quality-adjusted wages, we calculate the efficiency units of each occupation used in each establishment i at time t.¹¹ Results of these estimations can be found in Table A2.

3 Changes in the Spatial Organization of Firms

In Denmark in 2016, approximately 7% of all private-sector firms had more than one establishment. These ME firms accounted for around 47% of private-sector employment and 54% of total output. In comparison, the share of ME firms in 1981 was only 3.3%. Most of this increase has been driven by a rise in the number of ME firms, rather than a decline in the number of SE firms. These changes largely reflect shifts in the internal spatial organization of firms.

In Acosta and Lyngemark (2021), we document a significant increase in the spatial decentralization of firms, primarily along the extensive margin. Specifically, we find a 21% increase in the average number of establishments per firm and a 200% increase in the average distance between establishments and their headquarters (HQ) for firms in the manufacturing and service sectors between 1981 and 2016. In addition, the average share of employment located at firms' HQs declined by 13 percentage points, indicating a decentralization of employment away from HQs. However, that study remains silent on the fragmentation of tasks and on the mechanisms underlying these patterns.

The reallocation of jobs within ME firms could have important spatial implications. For example, an exogenous wage shock that increases the relative demand for high-skilled workers at HQs may contribute to the sorting of these workers into large cities–a phenomenon widely documented in the literature. In this paper, we complement the findings of Acosta and Lynge-

¹¹These efficiency units can be computed as $M_{ijt}^E = \sum_{p \in (i,m)} e^{X_{pmt}\beta^{in}}$ for managers, and $L_{ijt}^E = \sum_{p \in (i,l)} e^{X_{plt}\hat{\beta}^{l}}$ for workers, where $\hat{\beta}^c$ denotes the estimated parameters from equation 1 and $\sum_{p \in (i,c)}$ denotes the sum across all workers within an establishment and occupation.

mark (2021) by studying firm fragmentation along the intensive margin (changes in the relative composition of labor within establishments) and its relationship with wages.

We explore how managerial intensity has changed across establishments within firms by computing the ratio of managers to workers (M/L) at each establishment *i* in municipality *j* from firm *f* at time *t*, and estimate the following regression for ME firms:

$$\log\left(\frac{M}{L}\right)_{ijst} = \alpha_i + \delta_t + \lambda_{HQ_i,t} + \zeta_{js} + \varepsilon_{ijst},\tag{2}$$

where α_i are establishment fixed effects (FE), δ_t time FE, and $\lambda_{HQ_i,t}$ are HQ-time FE. We also control for commuting area*sector trends (ζ_{js}) to account for unobservable factors that could be driving these changes at the region and sector level. Standard errors are clustered at the municipality-year level to account for spatial and serial correlation. Figure 1 shows the estimated $\lambda_{HQ_i,t}$, which correspond to the differential changes in the manager-to-worker ratio at HQ relative to non-HQ establishments.

Figure 1: Ratio of Managers to Workers - HQ relative to non-HQ



This figure shows the HQ*year fixed effects from a regression of the log ratio of the establishments' managers to workers on establishment, year and HQ*year fixed effects and commuting area-sector trends. Estimates are relative to 1996. Dashed lines correspond to a 95% confidence interval.

This figure shows that between 1996 and 2011, the ratio of managers to workers increased by 22% at firms' headquarters (HQ) relative to non-HQ establishments. Although this ratio declined slightly following the financial crisis, it remained 13% higher in 2016 than in 1996. The changes are even more pronounced when measured in terms of efficiency units of labor: the ratio rose by approximately 24% between 1996 and 2011 and then stabilized at around 16% (Figure A1a in Appendix A). For robustness, we estimate the same regression using a balanced panel of establishments. As shown in Figure A1b, the change in this ratio is even larger in the balanced sample (30%), although the estimate is only marginally significant due to the smaller sample size and wider confidence intervals. We also show that the growth in managerial intensity is not driven solely by the growth of multinational firms by excluding firms that reported an establishment outside Denmark (Figure A1c), or by the commuting area-sector trends (Figure A1d). In both cases, the patterns remain consistent with our main findings. In a world composed only of SE firms, an increase in managerial intensity would typically reflect a decline in the relative cost of managers. However, firms operating across multiple locations can substitute labor not only within but also across establishments. Thus, their employment decisions also depend on the relative wages of both managers and workers across locations. To examine how wages have evolved within and across locations, we estimate two regressions. First, we regress the wage ratio of managers to workers (v/w) at HQ relative to non-HQ establishments, using the same set of fixed effects as in equation (2):

$$\log\left(\frac{v}{w}\right)_{ijst} = \alpha_i + \delta_t + \lambda_{HQ_{i,t}} + \zeta_{js} + \varepsilon_{ijst},\tag{3}$$

where wages correspond to the estimated quality-adjusted wages from equation (1) for both occupations. Figure 2a shows that this wage ratio has declined by approximately 3% over the sample period. Second, to examine how wages have changed across locations, we estimate the following regression for managers:

$$\log\left(\frac{v_h}{v_i}\right)_{jst} = \alpha_i + \delta_t + \zeta_{js} + \varepsilon_{it},\tag{4}$$

where v_i and v_h denote wages of managers at establishment *i* and its HQ *h*, respectively. The vector δ_t captures the average within-firm wage ratio over time, which we plot in Figure 2b. The figure shows that, since 1996, the average wage of managers at firms' HQs has increased by approximately 9% relative to those at non-HQ establishments. A similar regression for non-managers reveals that the wages of workers at HQs have also risen, though to a lesser extent: by around 2%.

Figure 2: Wage Differentials: Within and Across Locations



The left figure shows the HQ*year fixed effects from a regression of the log ratio of establishments' wages of managers to workers on establishment, year and HQ*year fixed effects. The right figure shows the time fixed effects from a regression of the log ratio of the establishments' wages of managers at the HQ to managers at a non-HQ establishment on establishment and year fixed effects. Both regressions include sector*region trends. Estimates are relative to 1996. Dashed lines correspond to a 95% confidence interval.

In the next section, we explore the role of wage changes within and across locations in driving the observed changes in specialization across establishments within ME firms. Our theoretical model incorporates both types of wage differentials and shows that, in response to exogenous wage shocks, ME firms substitute managers for workers not only *within*, but also *across* establishments. This substitution is shaped by internal communication costs and the role of intangible inputs in production. These mechanisms are novel and, to our knowledge, have not yet been explored in the existing literature.

4 Model

Motivated by the facts, we develop a partial-equilibrium model in which firms decide the number of establishments, together with the locations and labor composition of each. The model builds on the literature on multinational firms (Helpman, 1984; Horstmann and Markusen, 1987; Markusen, 2002), where firms must decide whether to become multinational, given the existence of trade costs and differences labor costs. As in this literature, our production technology features a firm-level public good, which gives rise to firm-level scale economies. However, there are decreasing returns to scale that create an incentive for establishing multiple plants. The complete mathematical derivations of our model can be found in section Appendix C.

Assume an economy with two locations $j \in \{o, s\}$, which can be thought of as the major city and a minor city or rural area, respectively. For simplicity, we assume that the firm's HQ's location is fixed at j = o. Firms in this economy produce a single final good (Y) using land, managers (m) and production workers (L). Moreover, firms also use headquarters services (H), which can be thought of as the different services produced at the firms' HQ and that can be used by the other establishments. These services include activities such as corporate and strategic planning, and tend to be manager-intensive activities (Aarland et al., 2007; Garicano, 2000). Atalay et al. (2014) show that these intangible inputs are crucial in explaining vertical integration and the formation of ME firms.

Due to the existence of communication costs between establishments, not every unit of H produced at HQ can be used in a particular establishment. Moreover, while managers can work towards the production of Y or H, workers can only work in producing the former. Given these technological assumptions, the firm first decides whether to have one or two establishments and the location of the potential second unit. In a second stage, the firm decides the number of managers and workers at each establishment. These decisions are driven by several forces. Wage growth in the central location create an incentive for the firm to open new establishments and substitute workers out of the HQ. However, by opening new establishments, firms incur extra fixed costs of production and communication costs, and could be missing out on some of the productivity advantages present at HQ. Finally, decreasing returns to scale in production causes centralized firms to be less profitable at the margin.

We assume that firms take the price of the final good (p), local land prices (r_j) , and local wages as given $(w_j$ for production workers and v_j for managers). Given the nature of this problem, we solve it using backward induction. First, we present the labor composition problem at the establishment level, taking the number of establishments, their locations and the amount of H. Based on this solution, we analyze the problem of a firm choosing the optimal amount of H. Finally, we study the firm's decision on the extensive margin.

4.1 The Problem of the Establishment

An establishment *i* produces output using both types of labor, land and *H*. Production of the final good in each establishment is Cobb-Douglas and requires production workers (L_i) , a managerial bundle (\mathbb{M}_{ij}) and one unit of land. In particular, the managerial bundle is a function of local managers and the amount of *H* received by the establishment.¹² Communication costs between establishments cause attenuation of *H*; thus, establishments may not receive the full amount of *H* produced by the HQ. We assume communication costs take the form of an iceberg cost: when HQ produces 1 unit of *H*, an establishment located in *s* receives $\tau \in [0, 1]$ units, while an establishment in *o* receives the full amount ($\tau_o = 1$). Final output also depends on a firm-specific productivity shock A_i . Moreover, we assume that managers are more productive when they are at the HQ; thus, their managerial input is augmented by $\mu_o > 1$.¹³ Summing up, an establishment *i* located in *j* solves the following maximization problem:

$$\max_{\{m_i, L_i\}} pY_{ij} - v_j m_i - w_j L_i - r_j \quad \text{s.t.} \quad Y_{ij} = AL_i^{\alpha} \mathbb{M}_{ij}^{\beta} = AL_i^{\alpha} \left[f(\mu_j m_i, \tau_j H) \right]^{\beta}, \tag{5}$$

where $\alpha, \beta, \in (0, 1)$ and $\alpha + \beta < 1$, $\mu_o > 1$ and $\mu_s = 1$; v_j is the wage of managers in j, w_j the wage of production workers in j and r_j denotes the price of land in j. The solution of this problem yields the demand for both types of labor as functions of prices, productivity parameters, communication costs and HQ services. From this solution, we also obtain the establishment's profit, $\pi_i(\tau_j H)$.

4.2 The Problem of the Firm

Given the establishments' profits, the firm chooses the amount of HQ services that maximizes its profits. We assume that H are produced using only HQ managers: $H = \mu_o m_h$, where m_h denotes the number of HQ managers producing H.¹⁴ This problem is equivalent to choosing the number of managers producing H that maximizes the sum of profits within the firm, net of their wages. Specifically, denoting \mathbb{E} as the set containing all of the firm's establishments, the problem can be written as:

$$\max_{\{m_h\}} \sum_{i \in \mathbb{E}} \pi_i(\tau_j \mu_o m_h) - v_o m_h, \tag{6}$$

To derive the model's labor demand equations, we must specify a functional form for the managerial bundle M_{ij} , i.e., the relation between production managers and HQ services. Intuitively, if both types of managers were complements, establishments that receive a high amount

 $^{^{12}}$ We use a Cobb-Douglas production function for tractability. Even though a more flexible function (e.g., CES) might be desirable, the combination of such a function with our non-linear problem of the firm leads to a set of first-order conditions from which we cannot obtain an analytic solution of the optimal labor demand for both types of workers in both locations.

 $^{^{13}}$ Figure A2 shows the structure of the model for a firm that has HQ in o and a establishment in s, respectively.

¹⁴This assumption can be relaxed to allow for H to be produced using both types of labor. Doing so does not change the model's predictions: it makes them less stark, but make the model more intractable.

of HQ services would need to hire more local managers-for example, satellite establishments may need more managers to process the information it receives from the HQ. On the other hand, if both types of managers were substitutes, establishments receiving a high amount of HQ services would need fewer local managers in order to produce the final output-for example, if some of the work done by local managers could be done by HQ managers at the same time for all of the establishments. As we show in Section 6, our empirical estimates suggest a complementary relation between local and HQ managers. In particular, we find that the elasticity of substitution of managers across locations fades through distance. This prediction is consistent with a fixed-proportions production function with a managerial bundle of the form $\mathbb{M}_{ij} = \min\{\lambda \mu_j m_i, \theta \tau_j H\}$, where λ and θ are technology-determined constants.¹⁵

We derive the full model in Section C.1 of the Online Appendix. The solution shows that SE firms hire more of both types of workers when there are greater productivity advantages at the HQ, lower input prices, or a higher price for the final good, which can be interpreted as a proxy for product demand. The case of ME firms yields more nuanced insights, which we summarize as follows. First, a reduction in communication costs ($\uparrow \tau$) increases the demand for both types of labor in the satellite establishment and raises firm profits. Second, higher wages reduce labor demand and firm profitability. Third, because the production of the final good at any establishment requires HQ services, labor demand in the satellite establishment depends not only on the local wages of managers and workers but also on the wage of HQ managers. An increase in the cost of HQ managers leads to a reduction in the demand for both managers and workers across all locations. Fourth, stronger decreasing returns to scale reduce both firm profits and factor demand

Finally, the firm chooses the number and location of its establishments. In the current model setting, this decision reduces to choosing between operating only one establishment, opening a second in location o, or opening a second in location s. Intuitively, a firm will open a satellite establishment in s if the marginal benefit from doing so exceeds the associated fixed cost. This marginal benefit reflects a trade-off between wage differentials, communication costs, and headquarters (HQ) productivity advantages. The next subsection explores the theoretical results of our model both on the intensive and extensive margins.

4.3 Model Predictions

In this subsection, we start by showing how the firm's relative labor demands across and within establishments change when wages and communication costs change. The derivations and proofs of these results are shown in section C.2 of the Online Appendix. For ease of presentation, let us assume that the firm locates its second establishment in j = s and define the total number of managers in each establishment as $M_1 = m_1 + m_h$ and $M_2 = m_2$.

¹⁵A Cobb-Douglas or a linear function would imply the opposite direction for the above prediction. In section C.4 of the Online Appendix, we present a version of the model using a Cobb-Douglas managerial bundle. Ideally, we would like to derive the model using a CES function, but due to the non-linear nature of our model, it quickly becomes intractable.

Labor composition within Establishments

Start by analyzing the relative labor demand in a SE firm:

$$\left(\frac{M_1}{L_1}\right)_{SE} = \frac{\beta}{\alpha} \frac{w_o}{v_o}.$$
(7)

This ratio depends only on the establishment's relative wage between managers and workers. Therefore, when a SE firm faces exogenous wage shocks, it can only respond by changing the relative composition of that establishment. This is not the case for establishments that belong to ME firms, where the ratio of managers to workers at non-HQ establishments given by:

$$\left(\frac{M_2}{L_2}\right)_{ME} = \frac{\beta}{\alpha} \frac{w_s}{v_s} \left(1 + \frac{v_o}{v_s} \frac{\lambda}{\theta \tau \mu}\right)^{-1}.$$
(8)

Just as in SE firms, this ratio depends negatively on the relative wages between managers and workers within the establishment. However, it also depends negatively on the price of managers at the HQ relative to the establishment. This implies that an exogenous increase in the price of HQ managers raises the cost of the managerial inputs used at non-HQ establishments, decreasing the demand for local managers. Finally, note that lower communication costs (τ) and higher HQ-productivity advantages (μ) magnify this effect.

Moreover, the ratio of manager to workers at the HQ is given by:

$$\left(\frac{M_1}{L_1}\right)_{ME} = \frac{\beta}{\alpha} \frac{w_o}{v_o} \left\{ 1 + \left[\left(\frac{\theta\tau}{\lambda}\right)^\beta \left(\frac{w_o}{w_s}\right)^\alpha \left(\frac{v_s}{v_o}\frac{\theta\tau\mu}{\lambda} + 1\right)^{\alpha-1} \right]^{\frac{1}{1-\alpha-\beta}} \right\}.$$
 (9)

In addition to the standard labor demand channel, this ratio depends on the wage gap of workers and managers between both locations, communication costs and HQ productivity advantages. This equation suggests that lower wages at a non-HQ establishment would lead to a higher manager-to-workers ratio at at the HQ. Moreover, lower communication costs magnify this effect, but also have an ambiguous effect on managerial intensity at the HQ. On one hand, since non-HQ establishments receive more H, there is an incentive to produce more HQ services and hire more managers at the HQ. On the other hand, if the establishment receives a higher quantity of HQ services, it also needs to hire more local managers, which would drive down total profits and the demand for H.

It is now evident that, when facing an exogenous wage shock, ME firms have an extra margin of adjustment that SE firms do not have: They can substitute labor across locations. Moreover, firms have a third margin of adjustment, coming from the scale effects generated by the public nature of H. To see this more clearly, consider an exogenous shock that renders managers in the central location more expensive. Using a Slutsky-type decomposition, the total effect of this shock on the manager-to-worker ratio at the HQ can be decomposed as the sum of three effects:

$$\frac{\partial M_1/L_1}{\partial v_o} = \frac{\partial (M_1/L_1)^c}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial v_o}$$

$$\frac{\partial M_1/L_1}{\partial v_o} = \underbrace{\underbrace{\partial (M_1/L_1)^c}_{\partial (v_o/w_o)} \cdot \frac{\partial (v_o/w_o)}{\partial v_o}}_{\text{Standard Subst. Effect}} + \underbrace{\underbrace{\partial (M_1/L_1)^c}_{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/w_o)} \cdot \frac{\partial (v_o/w_o)}{\partial v_o}}_{\text{Standard Scale Effect}} \cdot \frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o}, \quad (10)$$

$$+ \underbrace{\underbrace{\partial (M_1/L_1)^c}_{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o}, \quad (10)$$

$$+ \underbrace{\underbrace{\partial (M_1/L_1)^c}_{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o}, \quad (10)$$

where the superscript c denotes the conditional labor demand functions that result from the firm's cost minimization problem, and Y_1 and Y_2 correspond to the total amount of output produced at HQ and non-HQ establishments, respectively. In Section C.3 from the Online Appendix, we show the details of this decomposition.

The first two terms on the right-hand side are the standard substitution and scale effects from a regular Slutsky decomposition. On one hand, if managers are more expensive, the establishment can substitute managers for workers (substitution effect). On the other hand, since the production of final output requires both types of labor, higher managerial costs reduce output and thus total labor demand (scale effect). Both effects suggest a decline in managerial intensity at the HQ.

However, a third effect arises in ME firms. As HQ managers become more expensive, firms shift production to satellite establishments, where labor is cheaper. Yet, production in satellites requires HQ services. As satellite establishments grow, the demand for these HQ services rises, increasing the firm-wide demand for HQ managers. When the elasticity of substitution across locations is low and HQ services are important, this scale-induced increase in HQ labor demand can outweigh the direct price effect. Consequently, an exogenous rise in the price of HQ managers can lead to an increase in managerial intensity at the HQ. This counterintuitive prediction is a distinctive and testable implication of our model. We formalize these results in the next proposition.

Proposition 1. Managerial Intensity

(a) Higher relative wages at the establishment's location $(\uparrow \frac{v}{w})$ lead to less manager-intensive establishments $(\downarrow \frac{M}{L})$. This holds for every establishment in the economy.

(b) For multi-establishment firms, a wider worker wage gap across locations $(\uparrow \frac{w_o}{w_s})$ leads to a more manager-intensive HQ relative to the non-HQ establishment $(\uparrow \frac{M_h/L_h}{M_i/L_i})$.

(c) For multi-establishment firms, a wider managerial wage gap across locations $(\uparrow \frac{v_o}{v_s})$ leads to a more manager-intensive HQ relative to the non-HQ establishment $(\uparrow \frac{M_h/L_h}{M_i/L_i})$. Lower communication costs $(\uparrow \tau)$ magnify the effect.

Formal proof of all propositions are included in Section C.2 from the Online Appendix. We

can also derive the elasticities of the HQ manager-to-worker ratio with respect to a change in the wage gap of either workers or managers across locations:

Corollary 1. Keeping the within-establishment wage ratio $\left(\frac{v}{w}\right)$ constant and defining $M_1 = m_1 + m_h$, the elasticities of the HQ ratio of managers to workers, with respect to changes in the wage gap of workers and managers across locations are given by:

- (a) For workers: $\frac{\partial \log(M_1/L_1)}{\partial \log(w_o/w_s)} = \frac{\alpha}{1-\alpha-\beta} \cdot (1-\omega) > 0.$
- (b) For managers: $\frac{\partial \log(M_1/L_1)}{\partial \log(v_o/v_s)} = \frac{1-\alpha}{1-\alpha-\beta} \cdot (1-\gamma) \cdot (1-\omega) > 0.$

where $\gamma = \frac{v_o/\theta\tau\mu}{(v_s/\lambda)+(v_o/\theta\tau\mu)} > 0$ corresponds to the cost share of non-HQ managers on the total managerial costs of the non-HQ establishment, and $\omega = \frac{v_o m_1}{v_o M_1} > 0$ corresponds to the cost share of production managers on total managerial costs at the HQ.

Labor composition across Establishments

We also derive two equations that help us understand the substitution patterns of workers and managers across establishments. Empirically, these two equations are useful for two reasons. First, these equations yield relatively simple linear regressions, so their reduced-form estimates will help us test some of the model's predictions. Second, the across-location elasticities of substitution implied by these equations provide important identifying variation to estimate the structural parameters of our model. In particular, we can write the ratio of workers and managers at the HQ relative to the non-HQ establishment as

$$\frac{L_1}{L_2} = \left[\left(\frac{w_o}{w_s} \right)^{\beta-1} \left(\frac{v_s}{v_o} \mu + \frac{\lambda}{\theta \tau} \right)^{\beta} \right]^{\frac{1}{1-\alpha-\beta}},$$
(11)

$$\frac{M_1}{M_2} = \frac{\lambda}{\theta \tau \mu} + \frac{1}{\mu} \left[\left(\frac{w_o}{w_s} \right)^{-\alpha} \left(\frac{v_s}{v_o} \mu + \frac{\lambda}{\theta \tau} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha-\beta}}.$$
(12)

From these expressions, we highlight three key effects. First, lower communication costs reduce the ratio of both workers and managers at the HQ relative to the non-HQ establishment. With improved communication, non-HQ establishments receive more services from the HQ and expand production, hiring more managers and workers. Second, an increase in HQ manager-specific productivity (μ) raises the ratio of workers at the HQ relative to the non-HQ establishment, but its effect on the managerial ratio is ambiguous. On one hand, higher productivity at the HQ increases the marginal value of HQ managers, creating an incentive to employ more managers at the HQ. On the other hand, this productivity gain also boosts the production of HQ services, which raises demand for local managers at the non-HQ establishment as it scales up its output. Third, the share of both workers and managers at the HQ decreases with the respective wage gap across locations. This result implies that the relative size of the HQ declines when the wage differential between the HQ and the non-HQ establishment widens.

From equations (36) and (37), we compute the across-location elasticities of substitution for

workers and managers as:

$$\frac{\partial \log(L_1/L_2)}{\partial \log(w_o/w_s)} = \frac{\beta - 1}{1 - \alpha - \beta} < 0,$$

$$\frac{\partial \log(M_1/M_2)}{\partial \log(v_o/v_s)} = \frac{\alpha - 1}{1 - \alpha - \beta} \cdot (1 - \gamma) \cdot \omega < 0,$$
 (13)

where $M_1 = m_1 + m_h$.

While the elasticity of substitution of workers across establishments is constant, this is not the case for managers. The latter elasticity depends on two cost shares: (i) the cost share of non-HQ managers on the total managerial cost of the non-HQ establishment, and (ii) the cost share of production managers on the total managerial costs at the HQ. If production did not depend on HQ services, the elasticity would converge to a constant given by $\alpha - 1/1 - \alpha - \beta$. Moreover, as the use of HQ services increases, this elasticity converges to zero. Therefore, this elasticity implies that when firms are highly dependent on the within-firm public good, we would observe a low degree of substitution of managers across establishments for a given exogenous change in the wage gap of managers. Furthermore, if $\beta < \alpha$, the across-location elasticity of substitution for workers will always be more negative than the one for managers. These results can be summarized as:

Proposition 2. Substitution Across Establishments

(a) A steeper wage gap for workers ($\uparrow \frac{w_o}{w_s}$) leads to a lower share of workers at the HQ ($\downarrow \frac{L_h}{L_i}$).

(b) A steeper wage gap for managers ($\uparrow \frac{v_o}{v_s}$) leads to a lower share of managers at the HQ ($\downarrow \frac{M_h}{M_i}$). Lower communication costs ($\uparrow \tau$) magnify the effects.

Finally, our model also provides us with the following result on the firm's decision regarding its number of establishments:

Proposition 3. Firms are more likely to open a satellite establishment with a decrease in communication costs, or an increase in the price of inputs at the HQ (for either type of labor or land), productivity advantages at the HQ, or firm-specific productivity.

5 Empirical Strategy

The model developed in the previous section delivers a series of testable predictions, along with a system of labor demand equations that can be estimated using establishment-level data to investigate the mechanisms underlying the observed increase in HQ managerial intensity. This section outlines the empirical strategy we follow to pursue both objectives. In line with the model, we use subindex i to denote an establishment and h to denote its HQ. Subindices j and k refer to the municipality of the establishment and HQ, respectively, while t indicates the year, and s the sector. For instance, M_{ijhst} represents the number of managers in establishment i, located in municipality j, with HQ h, in sector s, in year t.

5.1 Reduced Form Estimation

We begin by estimating reduced-form regressions to test the model's predictions and assess its validity. Specifically, we estimate the following relative labor demand equations to test Proposition 1:

$$d\log\left(\frac{M}{L}\right)_{ijhst} = \psi_1 d\log\left(\frac{v}{w}\right)_{it} + d\delta_t + \zeta_{js} + d\epsilon_{ijhst}, \tag{14}$$

$$d \log \left(\frac{M_h/L_h}{M_i/L_i}\right)_{jst} = \psi_2 d \log \left(\frac{w_h}{w_i}\right)_t + d\delta_t + \zeta_{js} + d\epsilon_{ijhst},$$

$$d \log \left(\frac{M_h/L_h}{M_i/L_i}\right)_{jst} = \psi_3 d \log \left(\frac{v_h}{v_i}\right)_t + d\delta_t + \zeta_{js} + d\epsilon_{ijhst},$$
 (15)

where $d\delta_t$ denotes year fixed effects, ζ_{js} captures commuting area-sector trends, and $d\epsilon_{ijhst}$ is the error term, which we cluster at the municipality*year level in all regressions. Proposition 1 predicts that $\psi_1 < 0$ for all types of establishments. Therefore, we also estimate equation (14) with an interaction between wage changes and establishment type (SE, HQ of ME, non HQ of ME). To further explore heterogeneity, we estimate an additional specification that interacts wage changes with the distance between an establishment and its HQ. Lastly, Proposition 1 also predicts that $\psi_2 > 0$ and $\psi_3 > 0$, implying that larger wage gaps between the HQ and other establishments lead to more manager-intensive HQs.

To test Proposition 2, we estimate the following two equations:

$$d\log\left(\frac{L_h}{L_i}\right)_{jst} = \psi_4 d\log\left(\frac{w_h}{w_i}\right)_t + d\delta_t + \zeta_{js} + d\eta_{ijhst},\tag{16}$$

$$d\log\left(\frac{M_h}{M_i}\right)_{jst} = \psi_5 d\log\left(\frac{v_h}{v_i}\right) + d\delta_t + \zeta_{js} + d\nu_{ijhst}.$$
(17)

Our model predicts that $\psi_4 < \psi_5 < 0$. Moreover, Proposition 2 suggests that higher communication costs should reduce the elasticity of substitution across locations, particularly for managers. To test this, we also estimate both specifications with an interaction between wage changes and the distance between the establishment and its HQ. Finally, to examine whether agglomeration economies influence these relationships, we interact wage changes with a binary variable for whether a firm's HQ is located in the Copenhagen metropolitan area.¹⁶

Since observed wages at each establishment are equilibrium outcomes, identifying the causal parameters in the previous equations requires a source of exogenous variation in changes to the relative wages of managers and non-managers, both within and across locations. In the following section, we introduce a framework that generates exogenous labor supply shocks faced by establishments, which we use as instruments for wage changes.

 $^{^{16}}$ Eckert et al. (2022) show that both wages and returns to experience are significantly higher in Copenhagen relative to the rest of the country. This supports the use of a Copenhagen dummy variable as a proxy for agglomeration.

5.2 Identification Strategy: Labor Supply Shocks

Given the simultaneity bias in the estimation of labor demand equations, we use local labor supply shocks to recover the parameters of interest. We follow the approach proposed by Card (2001)–and used in Lewis (2011) and Baum-Snow et al. (2018)–to build labor supply shocks using immigration share-shifters to instrument changes in the wages paid by establishments to both managers and non-managers. The idea behind this type of instrument is that immigrants are more likely to live in municipalities with a relatively high number of immigrants from their country of origin, regardless of local labor market conditions, mostly due to the possibility of stronger social networks, but also due to relatively higher amenities.¹⁷

We start by building a standard immigration share-shifter for both managers and workers. Define IS_{ojt}^R and IS_{ojt}^F as the immigration shocks for type of worker o at the respective residence (R) and workplace (F) municipality j in year t. We construct our instrument in two steps. First, define the immigration shock at the residential municipality as:

$$IS_{ojt}^{R} = \sum_{c} \frac{L_{ocjt^{0}}}{L_{oct^{0}}} \cdot \log Imm_{o,c,DK-j,t},$$
(18)

where L_{ocjt^0} denotes the number of workers of occupation o from country c in municipality of residence j in a base year t^0 ; L_{oct^0} denotes the total number of workers in Denmark in occupation o from country c in t^0 . Finally, $\log Imm_{o,c,DK-j,t}$ denotes the logarithm of the total stock of immigrants of occupation o from country c in year t, leaving municipality j out.

Since immigrant enclaves are not necessarily located in the same municipalities as establishments, immigration shocks at the residential municipalities may not be too relevant from the establishments' perspective as they may not shift labor supply curves in their municipality. In fact, due to differences in commuting between municipalities, a labor supply shock in one municipality might disperse geographically into other municipalities. For example, the labor supply that establishments experience in a municipality with high employment density, such as Aarhus, will be affected not only by the "residential" immigration shocks in Aarhus, but also by the shocks happening in nearby municipalities, such as Favrskov or Silkeborg.

Therefore, we augment the standard immigration share-shifter by mapping these shocks from municipalities of residence into workplace municipalities using historical commuting flows for Danish native workers in occupation o, π^D_{ojk,t^0} . Note that we use occupation-specific commuting shares to capture possible differences in commuting costs between managers and workers. More specifically, our instrument is defined as

$$IS_{okt}^F = \sum_j IS_{ojt}^R \cdot \pi_{ojk,t^0}^D.$$
⁽¹⁹⁾

We use these commuting-augmented immigration shocks as instruments for changes in establishment-

¹⁷Insofar as social networks tend to consist of individuals with similar characteristics, it is important to also consider other treats that may influence labor market outcomes, such as skill level and occupation, among others (Portes and Manning, 2019).

level wages for both managers and workers:

$$d \log v_{ijhst} = a_1 dIS_{M,jt}^F + d\delta_t + \zeta_{js} + de_{ijhst},$$

$$d \log w_{ijhst} = a_2 dIS_{L,jt}^F + d\delta_t + \zeta_{js} + de_{ijhst},$$

where $\log v_{ijhst}$ and $\log w_{ijhst}$ respectively represent the logarithm of the wage of managers and workers in establishment *i* located in *j*, as estimated in Section 2. Columns (1) and (2) of Table 1 present the results of these two estimations. Column (2) shows that establishments in municipalities with larger predicted immigration flows saw a reduction in wages for nonmanagers, as expected. Although the estimated coefficient for managers in Column (1) is positive, the first-stage F-statistic is weak; hence, our identifying variation primarily stems from labor supply shocks affecting non-managers.¹⁸

Table 1: First Stage: Establishment-level Wages and Labor Supply Shocks

	(1)	(2)	(3)	(4)	(5)
Dependent variable	$\log v_i$	$\log w_i$	$\log\left(\frac{v}{w}\right)_i$	$\log\left(\frac{w_h}{w_i}\right)$	$\log\left(\frac{v_h}{v_i}\right)$
	_	_			
Instrument:	IS_{Mj}^F	IS_{Lj}^F	$IS_{Mj}^F - IS_{Lj}^F$	$IS_{Lk}^F - IS_{Lj}^F$	$IS_{Mk}^F - IS_{Mj}^F$
Estimate	0.0006**	-0.0010***	-0.0019***	0.0001***	0.0010***
	(0.0003)	(0.0001)	(0.0003)	(0.000)	(0.0001)
Ν	250,774	250,774	250,774	39,162	39,162
Kleibergen-Paap rk Wald F	4.15	74.28	30.34	34.61	63.75

Notes: This table shows the results of regressions of establishment-level wages on the respective commutingaugmented immigration shock. As dependent variables, column (1) uses only managers, column (2) only workers, column (3) the ratio of managers to workers within establishment, column (4) the ratio of workers at the HQ relative to non-HQ, column (5) the ratio of managers at the HQ relative to non-HQ. j and k correspond to the location of the establishment and its HQ, respectively. Columns (4) and (5) only use non-HQ establishments. Standard errors clustered by establishment's municipality and year. All regressions include establishment and year fixed effects and sector*commuting area pre-trends. *** p < 0.01, ** p < 0.05, * p < 0.1.

Since our estimations rely on relative labor supply shocks, we use different combinations of the previous first stage equations: (i) the immigration shock of managers relative to workers in an establishment's municipality as an instrument for changes in the relative wage between managers and workers; (ii) the immigration shock of workers at the HQ location relative to the establishment's municipality as an instrument for changes in the relative wage of workers across locations; and (iii) similarly for managers. These equations can be written as:

$$d \log \left(\frac{v}{w}\right)_{ijhst} = b_1 d(IS^F_{Mjt} - IS^F_{Ljt}) + d\delta_t + \zeta_{js} + de_{ijhst},$$

$$d \log \left(\frac{w_h}{w_i}\right)_{jst} = b_2 d(IS^F_{Lkt} - IS^F_{Ljt}) + d\delta_t + \zeta_{js} + de_{ijhst},$$

 $^{^{18}}$ This variation is expected, as a large share of immigrants to Denmark come from countries with lower average education levels and are unlikely to enter as managers. In fact, only an average of 7.2% of foreigner workers are employed as managers each year in Denmark and 3 out of 5 foreigner workers come from countries outside the European Union or other high-income countries.

$$d\log\left(\frac{v_h}{v_i}\right)_{jst} = b_3 d(IS^F_{Mkt} - IS^F_{Mjt}) + d\delta_t + \zeta_{js} + de_{ijhst},$$
(20)

where *i* denotes the establishment, *h* their HQ, and *j* and *k* their respective locations. Columns (3) to (5) from Table 1 present the estimates of b_1 , b_2 , and b_3 , respectively. These three estimates are statistically significant and exhibit F-statistics above 10 in all cases. These first-stage expressions can be interpreted as relative labor supply equations complementing our relative labor demand system from equations (20)-(20), which we estimate using two-stage least squares (2SLS).

We believe our instrument is valid in our setting for four reasons. First, the share of employment held by immigrants in Denmark has been increasing constantly from a 1% in 1994. Using a similar instrument, Foged and Peri (2016) show that municipalities with a relatively large exposure to immigration experienced significant changes in the supply of low-skilled labor. Second, there is relevant exogenous variation across immigrants' countries of origin and time of arrival (Foged and Peri, 2016). Third, due to differences in the accession dates of different countries within the European Union, there is also variation in terms of sector, type of workers and origin of people who could enter Denmark without restrictions (Fackler, 2018). Fourth, Denmark implemented a quasi-random dispersal policy between 1986 and 1998 with the goal of distributing refugees across municipalities in proportion to population size and available public housing (Damm, 2009). This policy generated national clusters of refugees that were independent of local labor market conditions (Eckert et al., 2022). Thus, we choose the period between 1986 and 1994 as our base period, when the dispersal policy was at its full strength.

Econometrically, for our instrument to be valid, it must be that predicted immigration flows are not correlated with unobserved factors driving changes in firms' labor demand decisions. That is, we must assume that manager and non-manager immigrants arriving in Denmark before 1994 could not anticipate which firms were going to generate new jobs afterward, nor they generated differential job growth subsequently. Specifically, consistent identification of the model's parameters requires that $E[\mathbf{dIS}_{\mathbf{t}}^{\mathbf{F}} \cdot d\varepsilon_t] = 0$, where $\mathbf{dIS}_{\mathbf{t}}^{\mathbf{F}} = \{dIS_{olt}^F\}_{\forall(o,l)}$, and $d\varepsilon_t$ is the vector containing the residuals from the relative labor demand regressions.

Finally, our identification strategy also relies heavily on the panel structure of our data. In all of our estimations, we include establishment and year fixed effects and trends at the establishment's sector*commuting area. Establishment fixed effects allow us to control for unobservable time-invariant establishment characteristics that might alter the establishments' relative labor demand decisions. Therefore, our identification comes from changes in relative labor demand decisions within and across locations, not from levels. In addition, sector*commuting area trends allow us to control for unobserved trends that affect relative labor demand decisions in a particular sector-region pair, such as new infrastructure or technologies. In Table A4 of the Online Appendix, we present different robustness checks for the first stage estimation. Besides the baseline estimates (Panel A), we estimate the regression building the IV using 1994 shares (Panel B), excluding sector*area trends (Panel C), and excluding Denmark's four largest cities (Panel D). Results are highly similar across all specifications.

5.3 Structural Model Estimation

From our structural model, we also derive a system of relative labor demand equations: one equation for relative labor demand *within* establishments, and two for relative labor demand *across* establishments. We estimate the model using the two *across* equations, and leave the *within* equation to test the model fit and perform a decomposition. Before doing this, we need some additional structure.

First, due to the lack of detailed data on communication costs, we assume that they depend negatively on the distance between two locations and a time-varying elasticity parameter: $\tau_{jk} \equiv dist_{jk}^{-c_t}$. Therefore, we infer communication costs out of changes in the estimated parameter c_t , which captures the percentage change in communication costs given by a 1% change in the distance to HQ (i.e., the distance elasticity). Specifically, changes over time in the effect of distance (between establishments and their HQ) on labor demand decisions will be the main source of identifying variation for this parameter.

Second, consider the *across* equation for workers. Assume that the demand for workers at HQ relative to an establishment consists of three components: (i) the total number of workers at establishment *i* and its HQ *h*, determined by the firm's maximization problem, denoted as L_{ijst} and L_{hst} ; (ii) a multiplier e^{δ_t} that captures economy-wide shocks affecting all establishments in a year; and (iii) a multiplier $e^{\eta_{ijhst}}$ that captures unobserved, time-varying factors influencing the establishment's relative demand for workers across locations. Assume a similar structure for the relative demand for managers across locations, with an error term $e^{\nu_{ijhst}}$. Given this structure, we differentiate both equations in logs with respect to wages and communication costs to obtain:

$$d\log\left(\frac{L_h}{L_i}\right)_{jst} = \frac{\beta - 1}{1 - \alpha - \beta} d\log\left(\frac{w_h}{w_i}\right)_t - \frac{\beta(1 - \gamma_i)}{1 - \alpha - \beta} d\log\left(\frac{v_h}{v_i}\right)_t + \frac{\beta\gamma_i}{1 - \alpha - \beta} \sum_t \mathbf{1}_t \cdot [c_t \log dist_{ih}] + d\delta_t + \zeta_{rjs} + d\eta_{ijhst},$$
(21)

$$d\log\left(\frac{M_h}{M_i}\right)_{jst} = \frac{(\alpha-1)(1-\gamma_i)\omega_h}{1-\alpha-\beta}d\log\left(\frac{v_h}{v_i}\right)_t - \frac{\alpha\omega_h}{1-\alpha-\beta}d\log\left(\frac{w_h}{w_i}\right)_t - \left[\frac{(\alpha-1)\gamma_i\omega_h}{1-\alpha-\beta} - (1-\omega_h)\right]\sum_t \mathbf{1}_t \cdot [c_t\log dist_{ih}] + d\delta_t + \zeta_{js} + d\nu_{ijhst}, \quad (22)$$

where γ_i and ω_h are the cost shares defined in Corollary 1, which we recover from observed data on employment and wages from each establishment and use their lagged value to avoid endogeneity.¹⁹

¹⁹We define the number of managers in HQ services as those categorized as "Directors and Chief Executives." We then define the number of managers in the production of the final good at the HQ, m_h^y , as the difference

To recover our structural parameters, we jointly estimate equations (21), (22), and labor supply equations (20) and (20) using feasible generalized simultaneous nonlinear least squares (FGNLS), which allows the errors to be correlated across equations. All these equations have the following form $dY = f(\phi, W)dX + d\varepsilon$, where $\phi = \{\alpha, \beta, \{c_t\}\}, X$ corresponds to the vector of endogenous variables and W to exogenous variables. As shown by Cai et al. (2006), after incorporating in the system of equations a first stage of the form $dX = b \cdot dZ + de$ and $E[dZ \cdot de] = 0$, the parameter vector ϕ can be identified and consistently estimated.²⁰

5.4 Model Fit and Decomposition

Our model also generates an expression for the demand for managers relative to workers *within* establishments, separately for the three types of establishments: SE firms, and HQ and non-HQ establishments within ME firms. Applying log-differentiation, we can express this relationship as:

$$d\log\left(\frac{M}{L}\right)_{ijhst} = -d\log\left(\frac{v}{w}\right)_{it} + \mathbf{1}_{\{HQ\}}\frac{\alpha(1-\omega_h)}{1-\alpha-\beta}d\log\left(\frac{w_h}{w_i}\right)_t + \left\{\mathbf{1}_{\{HQ\}}\frac{(1-\alpha)(1-\omega_h)(1-\gamma_i)}{1-\alpha-\beta} - \mathbf{1}_{\{N\}}\gamma_i\right\}d\log\left(\frac{v_h}{v_i}\right)_t - \left\{\mathbf{1}_{\{HQ\}}\frac{\beta-(1-\alpha)(1-\gamma_i)}{1-\alpha-\beta}(1-\omega_h) + \mathbf{1}_{\{N\}}\gamma_i\right\}\sum_t \mathbf{1}_t \cdot [c_t\log dist_{ih}],$$

$$(23)$$

where $\mathbf{1}_{\{HQ\}}$ and $\mathbf{1}_{\{N\}}$ are indicator variables that equal one if an establishment is the HQ or a non-HQ of a ME firm, respectively. Using data and the estimated parameters from equations (21) and (22), we compute each of the four components from equation (23).²¹ The sum of these components gives us the predicted change in the manager-to-workers ratio: $d \log \left(\frac{\hat{M}}{L}\right)_{iihst}$.

6 Results

In this section, we follow the same structure as in Section 5. We start by presenting reducedform estimates of the relative labor demand equations, which allow us to test our model's main theoretical results. These regressions are also useful to show the variation that identifies the structural parameters. We then present estimates of our structural parameters, with which we quantify the relative importance of each proposed mechanisms in explaining the observed increase in HQ managerial intensity.

between the total number of managers at the firm's HQ (M_h) and the total number of managers in HQ services. In Table A3 from the Online Appendix, we provide a ranking of sectors based on the firms' average proportion of executives to the total number of workers.

²⁰To improve the computational efficiency of the estimation, we apply the Frisch-Waugh-Lovell theorem, together with the method of alternating projections, to account for establishment and year fixed effects (Correia, 2018). Afterwards, we demean the resulting variables within each sector-commuting area pair to account for the sector-region trends.

²¹Since the dependent variable for an HQ establishment depends on the wage gap between the HQ and a non-HQ establishment, we use the firm's largest non-HQ establishment when the firm has more than two establishments.

6.1 Reduced-Form Estimates

We start by testing the predictions from Proposition 1 by estimating equations (14) and (15), along with their corresponding first stage equations. Table 2 reports the estimates of withinestablishment wage elasticities. Column (1) presents results from the "standard" relative labor demand equation, with its coefficient captures the average elasticity of substitution between managers and non-managers within an establishment. The estimates suggest that a 1% increase in the relative wage of managers leads to a 0.94% decrease in the manager-to-worker ratio, consistent with the prediction from Proposition 1. We have not found any other estimates of the elasticity of substitution between managers and workers in the literature.

In Column (2), we estimate the elasticity of substitution separately for three types of establishments: SE firms, and HQ and non-HQ establishments in ME firms. The results suggest that the elasticity is larger in SE firms than in ME firms. Specifically, we find an elasticity of -1.14 in SE firms, while estimates for establishments in ME firms are not significantly different from zero. This implies that when an exogenous shock affects the relative wage of one type of workers, ME firms do not substantially adjust by substituting managers for workers within establishments, likely because they can reallocate labor across establishments, unlike SE firms. Finally, Column (3) suggests that the elasticity of substitution between managers and nonmanagers decreases with the establishments' distance to its HQ, implying less substitutability of labor in more distant establishments.

We test the second part of Proposition 1 in Columns (4) and (5). In line with our model, estimates show that larger wage gaps between HQ and the establishment can lead to a more manager intensive HQ.²² Our results indicate that a 1% increase in the wage of workers at the HQ relative to the non-HQ establishment leads to a 5.6% increase in the relative manager-to-worker ratio at the HQ; the corresponding elasticity for managers is 1.59%. Although the estimated elasticity is larger for workers than for managers, spatial wage changes have been at least four times greater for managers, suggesting that the overall effect is stronger in that case. Recall that this effect is a result of the firm scale effects brought by the non-rival property of HQ services.

In Table 3, we explore how the allocation of workers across establishments responds to exogenous changes in the spatial wage gap of each type of worker. These relationships allow us to test Proposition 2 from our model. Column (1) from Panel A shows the elasticity of substitution for workers between establishments and their HQ. Column (4) from Panel B shows the equivalent elasticity for managers. To the best of our knowledge, these are the first estimates of the elasticity of substitution across locations of ME firms.

In particular, Column (1) indicates that if wages of workers at the HQ relative to a non-HQ establishment increase by 1%, the relative number of workers at the HQ decreases by almost 10% on average. For managers, the elasticity of substitution across locations is of 1.1%. These

 $^{^{22}}$ The number of observations is substantially lower as Columns (4) and (5) only consider non-HQ establishments from ME firms, while Columns (1) to (3) consider all establishments in the data.

Dependent variable:	$(1) \\ d\log\left(\frac{M}{L}\right)_i$	$(2) \\ d\log\left(\frac{M}{L}\right)_i$	$(3) \\ d\log\left(\frac{M}{L}\right)_i$	$ \begin{pmatrix} (4) \\ d \log \left(\frac{M_h/L_h}{M_h/L_h} \right) $	(5) $d\log\left(\frac{M_h/L_h}{M_*/L_*}\right)$
-	0 (L) 1	0 (L) 1	0 (L) i	(M_i/L_i)	$\sim (M_i/L_i)$
$d\log\left(\frac{v}{w}\right)_i$	-0.942**		-1.436***		
	(0.378)		(0.434)		
x Single-Est		-1.141***			
x HQ Multi-Est		(0.381) -0.644			
		(0.433)			
x non HQ Multi-Est		0.510			
Log(Dist IIO)		(0.473)	0.000		
$\log(\text{Dist HQ})$			(0.000)		
$d\log\left(\frac{v}{2}\right) * \log(\text{Dist HQ})$			0.339***		
			(0.075)		
$d\log\left(\frac{w_h}{w_i}\right)$				5.566^{***}	
				(1.454)	
$d\log\left(\frac{v_h}{v_i}\right)$					1.591^{***}
					(0.379)
Observations	250,774	250,774	250,774	39,162	39,162

Table 2: Relative Labor Demand Estimation Within-Establishment Wage Elasticities

Notes: This table shows the results from different regressions of the establishment's manager to worker ratio at the left hand side $d \log \left(\frac{M}{L}\right)$ as a function of their relative wage $d \log \left(\frac{v}{w}\right)$, distance to their HQ and a variable that equals 1 if their HQ is located in Copenhagen. Standard errors clustered by establishment's municipality and year in parentheses. All regressions include establishment and year fixed effects, and sector*commuting area pre-trends. *** p < 0.01, ** p < 0.05, * p < 0.1

results indicate that on average, firms respond more to spatial differences in the wages of workers than to differences in managerial wages. This would be in line with the results of our model—in particular, equation (38), which suggests a larger elasticity of substitution for workers than for managers.

Our model also suggests that the elasticity of substitution for managers across establishments varies with distance from HQ, while for workers, this elasticity remains constant. To investigate this, we estimate the same regressions but include an interaction term between the distance to the HQ and the respective wage ratio. Columns (2) and (5) display the results for workers and managers, respectively. These results show that the elasticity of substitution between HQ and non-HQ establishments decreases as the distance between them increases, with the effect being significant for managers but not for workers.

The results presented in Column (5) indicate that firms respond more to changes in the managerial wage gap for establishments that are closer to the HQ compared to those that are farther away. For instance, consider a firm with two non-HQ establishments that experiences an exogenous reduction in managerial wages (relative to the HQ) at both locations. This firm is more likely to increase the relative size of the closer establishment than the more distant one. This could be due to the easier monitoring of nearby establishments or the logistical ease of

transferring employees to closer locations. As shown in Figure A3 in the Online Appendix, the elasticities for both managers and workers are negative and significantly different from zero, even for establishments located up to 500 km from their HQ–the maximum distance between municipalities in Denmark.

Panel A: Workers	(1)	(2)	(3)
Dependent variable:	$\log\left(\frac{L_h}{L_i}\right)$	$\log\left(\frac{L_h}{L_i}\right)$	$\log\left(\frac{L_h}{L_i}\right)$
	. ,	. ,	
$d\log\left(\frac{w_h}{w_h}\right)$	-0.802***	_19 11***	-0 006***
$u \log \left(w_i \right)$	-3.002	(2, 401)	(1, 667)
	(0.184)	(2.401)	(1.007)
Log(Dist to HQ)		-0.007**	
		(0.003)	
$d\log\left(\frac{w_h}{w_h}\right) * \text{Log}(\text{Dist to HQ})$		0.483	
$\left(\begin{array}{c} w_i \end{array} \right) \in \operatorname{Log}(2 \text{ iso to } \Pi \mathcal{Q})$		(0.300)	
		(0.590)	
HQ in CPH			-0.001
·			(0.007)
$d \log \left(\frac{w_h}{w_h}\right) * HQ$ in CPH			0.132
$\left(\left(w_{i}\right) \right) $			(1.245)
			(11-10)
Observations	39,162	39,162	39,162
Panel B: Managers	(4)	(5)	(6)
Dependent variable	$\log\left(\frac{M_h}{M_h}\right)$	$\log\left(\frac{M_h}{M_h}\right)$	$\log\left(\frac{M_h}{M_h}\right)$
	$\log(M_i)$	$\log(M_i)$	$\log(M_i)$
$d\log\left(\frac{v_h}{v_i}\right)$	-1.082***	-2.223***	-1.801***
	(0.279)	(0.641)	(0.404)
$L_{og}(Dist to HO)$		0.015***	
Log(Dist to HQ)		(0.013)	
$d\log\left(v_{h}\right) + \log(\text{Dist to HO})$		0.020**	
$u \log\left(\frac{1}{v_i}\right) * \log(\text{Dist to IIQ})$		0.230	
		(0, 110)	
		(0.110)	
HQ in CPH		(0.110)	0.004
HQ in CPH		(0.110)	0.004 (0.006)
HQ in CPH $d \log \left(\frac{v_h}{r}\right) *$ HQ in CPH		(0.110)	0.004 (0.006) 0.810^{**}
HQ in CPH $d\log\left(\frac{v_h}{v_i}\right) * \text{HQ in CPH}$		(0.110)	0.004 (0.006) 0.810^{**} (0.319)
HQ in CPH $d\log\left(\frac{v_h}{v_i}\right) * \text{HQ in CPH}$		(0.110)	0.004 (0.006) 0.810^{**} (0.319)

Table 3: Relative Labor Demand EstimationAcross-establishment Wage Elasticities

Notes: This table shows the results from different regressions of the ratio of workers (Panel A) or managers (Panel B) at the HQ, relative to non-HQ establishments as a function of their respective relative wage, distance to their HQ and a variable that equals 1 if their HQ is located in Copenhagen. Standard errors clustered by establishment's municipality and year. All regressions include establishment and year fixed effects and sector*commuting area pre-trends. *** p < 0.01, ** p < 0.05, * p < 0.1.

Some of the results in Tables 2 and 3 could be compromised if our immigration instruments were correlated with the distance between establishments and their HQ. This would be problematic, as the instrument could then act as a shock to both relative wages and spatial distance, thereby violating the exogeneity condition required for identification. To address this concern, we examine the correlation between establishment-HQ distance and the instrumental variables. As shown in Table A5 in the Appendix, these correlations are not statistically different from zero. This finding suggests that the labor supply shocks we exploit are large enough to affect firms' labor demand decisions, but not large enough to influence extensive margin choices—such as opening or relocating establishments—which typically involve sizable fixed costs.

Finally, in Columns (3) and (6), we introduce an interaction term between wage gaps and the HQ-at-Copenhagen dummy to assess whether the elasticities of substitution are influenced by HQ-specific productivity advantages. The point estimates suggest that the elasticity of substitution for managers is lower for firms with HQ located in the Copenhagen labor market area, while there is no difference for workers. More specifically, when firms with HQ in Copenhagen experience an exogenous shock that increases the wages of HQ managers, they may be less inclined to substitute managers across establishments. This could be due to significant productivity advantages associated with maintaining HQ managers within the Copenhagen area. However, as shown in Table A5, the correlations between the Copenhagen dummy and the instruments are not statistically different from zero. Given the lack of a suitable identification strategy, these estimates should not be interpreted as causal.

We are also interested in knowing how the coefficients from the interactions between relative wages and distance to the HQ have changed over time, since this variation will identify the distance elasticity parameters. In Table 4, we present the results from the within- and acrossestablishment wage elasticities, allowing the interaction term to vary over four periods: 1996-2000, 2001-2005, 2006-2010, and 2011-2016. Estimates from Column (1) suggest that the effect of distance from the HQ on the elasticity of substitution between managers and workers has been fading over time. Results from Columns (2) and (3) indicate that over time, ME firms seem to be less willing to substitute workers or managers between HQ and non-HQ establishments that are relatively far from the HQ.

The positive interaction between wage changes and distance, along with its increase over time, provides evidence of a complementary relationship between local managers and lower communication costs. In our model, this relationship supports the use of a fixed-proportions production function for the managerial bundle. These results suggest that, in the case of an exogenous wage shock that makes two satellite locations cheaper, firms would be more inclined to shift workers to those locations if communication costs are lower. In our model, these locations would receive more HQ services, thereby requiring more local managers. This finding contrasts with research by Gumpert et al. (2022), who argue that middle-managers at non-HQ establishments act as substitutes for CEO time. However, our result aligns with findings from Bresnahan et al. (2002) and Fort (2017), both of whom identify complementarities between technology and worker skill.

	(1)	(2)	(3)
	$d\log\left(\frac{M}{L}\right)_i$	$d \log \left(\frac{L_h}{L_i}\right)$	$d\log\left(\frac{M_h}{M_i}\right)$
$d\log\left(\frac{v}{w}\right)_i$	-1.527^{***}		
	(0.397)		
$d\log\left(\frac{w_h}{w_i}\right)$		-12.02^{***}	
		(2.403)	
$d\log\left(\frac{v_h}{v_h}\right)$			-2.154^{***}
			(0.637)
Log(Dist to HQ)	-0.001	-0.008**	-0.017***
	(0.001)	(0.003)	(0.002)
$d \log (RelW) * Log(Dist to HQ) * (1996-2000)$	0.481***	-0.773*	-0.136
	(0.120)	(0.468)	(0.128)
$d \log (RelW) * Log(Dist to HQ) * (2001-2005)$	0.536^{***}	0.078	0.264^{*}
	(0.114)	(0.516)	(0.140)
$d \log (RelW) * Log(Dist to HQ)*(2006-2010)$	0.245^{***}	1.052^{**}	0.312^{**}
	(0.094)	(0.466)	(0.128)
$d \log (RelW) *$ Dist. to HQ*(2011-2016)	0.210	1.047**	0.358***
	(0.136)	(0.442)	(0.119)
N	250,774	39,162	39,162

Table 4: Relative Labor Demand Estimation and Distance*Time

Notes: This table shows the results from regressions on within- and across-establishments relative labor demand on their respective relative wage $(d \log (RelW))$, distance to the establishment's HQ and their time-varying interaction. Standard clustered by establishment's municipality and year in parentheses; j and k denote the location of the establishment and its HQ. All regressions include establishment and year fixed effects and sector*commuting area pre-trends. *** p < 0.01, ** p < 0.05, * p < 0.1

6.2 Estimates of the Structural Model

This subsection presents the estimates of α , β and $\{c_t\}$ that result from estimating equations (21) and (22). Table 5 reports estimates of the production function parameters using counts of workers in Column (1) and efficiency units of labor in Column (2); both set of parameters are quite similar. We estimate α to be 0.652 and β to be 0.223. These estimates suggest strong evidence of decreasing returns to scale, since $\hat{\alpha} + \hat{\beta} = 0.875$ is significantly less than 1 at the 99% confidence level.

Column (1) also reports the estimates of the distance elasticity relative to the 1996-1997 value. Our results suggest that the distance elasticity has been increasing since 2008. In light of our model, these estimates would imply an increase in communication costs during the study period. Several factors could explain these changes. First, while technological advancements have lowered the cost of transmitting information, the complexity of the information firms must collect and communicate may have increased. Modern firms rely more heavily on data and technology than in the past (Brynjolfsson and McAfee, 2014). This information burden is likely even greater given the increasing spatial fragmentation of firms, as documented in (Acosta and

Lyngemark, 2021). However, further research is needed to explore the role of knowledge and technology within firms.

Second, recall that these parameters are identified from variation in establishment distance and over time. In Table 4, we showed that the relationship between relative wages and labor demand across locations has increased, potentially driving the rising distance elasticities. Other factors, such as changes in transportation infrastructure, may also contribute to these trends. For example, the opening of the Great Belt Bridge (1998) and the Øresund Bridge (2000), which connected the Copenhagen area with the rest of Denmark and Sweden, respectively, could have significantly reshaped Denmark's economic geography by altering firms' internal organization.²³

The observed increase in HQ managerial intensity could also result from higher managerial productivity at HQ locations. Such a shock would raise both demand for managers and their wages over time. To test this alternative explanation, we estimate our model allowing the HQ-manager productivity parameter (μ) to vary over time. The results, shown in Column (3), are derived from an equivalent set of equations (21) and (22), detailed in Section C.5 of the Online Appendix. The parameter estimates remain largely unchanged. As expected, the managerial share of the production function decreases, and we observe a gradual rise in HQ managerial intensity. In the next section, we use estimates from Columns (1) to (3) in our structural decompositions.

Finally, we conduct two robustness checks of our model. First, we estimate a version that allows parameters to vary over time. The results (Table A6 in the Online Appendix) align with Table 5, but also show a steady decline in the non-managerial labor share and an increase in the managerial labor share over time. Second, we estimate the model excluding firms operating in a single labor market to ensure these firms are not driving our results. The results, presented in Table A7, closely resemble those in Column (1) of Table 5.

6.3 Decomposition of the Growth in HQ Managerial Intensity

Using the estimated parameters from Table 5, we start by corroborating the fit of our model by constructing the predicted ratio of managers to workers as given by equation (23), and comparing it to the observed increase in HQ managerial intensity shown in Section 3. The specific details of this test were discussed in Section 5.4. In Figure 3, we plot the observed (same as in Figure 1) and predicted series. The figure shows that our model closely replicates the observed changes in the manager-to-worker ratio over the full period, although it fails to capture some of the cyclical fluctuations in the series.

Using equation (23) we also investigate the extent to which changes in (i) within-establishment wage differences, (ii) across-establishment wage gaps and (iii) distance elasticity explain the predicted increase in HQ managerial intensity. Table 6 shows the results of this decomposition. Column (1) shows that only around 15% of the whole increase in HQ managerial intensity can

 $^{^{23}}$ Little research has examined the impact of these bridges. Notable exceptions include Mulalic et al. (2014) and Bütikofer et al. (2022).

be explained by changes in the relative wages of managers to workers within establishments (the "standard" labor demand channel). On the other hand, changes in distance elasticity are

Danamatan	Description	(1)	(2)	(3) Counts
Parameter	Description	Counts	En. Units	Counts
0	Labor Shara, Cabb Douglas	0 659***	0 607***	0 682***
α	Labor Share-Cobb-Douglas	(0.052)	(0.007)	(0.033)
в	Managerial Share–Cobb-Douglas	0.223***	0.270^{***}	0.194^{***}
٣		(0.048)	(0.059)	(0.039)
c_1	Distance Elasticity 1996-97	0.003***	0.002***	0.008***
		(0.001)	(0.001)	(0.001)
c_2	Distance Elasticity 1998-99 (Rel. c_1)	0.000	-0.000	0.001
		(0.003)	(0.003)	(0.004)
c_3	Distance Elasticity 2000-01 (Rel. c_1)	0.001	-0.001	0.000
		(0.004)	(0.003)	(0.004)
c_4	Distance Elasticity 2002-03 (Rel. c_1)	-0.007	-0.009**	-0.008
		(0.004)	(0.004)	(0.005)
c_5	Distance Elasticity 2004-05 (Rel. c_1)	-0.006	-0.009**	-0.007
		(0.005)	(0.004)	(0.006)
c_6	Distance Elasticity 2006-07 (Rel. c_1)	-0.001	-0.005	-0.004
		(0.005)	(0.004)	(0.006)
c_7	Distance Elasticity 2008-09 (Rel. c_1)	0.018***	0.012***	0.017***
		(0.005)	(0.004)	(0.006)
c_8	Distance Elasticity 2010-11 (Rel. c_1)	0.025^{***}	0.019^{***}	0.025^{***}
_	Distance Electicity 2012 12 (Del)	(0.005)	(0.005)	(0.000)
c_9	Distance Elasticity 2012-15 (Ref. c_1)	(0.029^{+++})	(0.025^{++})	$(0.027)^{1}$
0	Distance Flasticity 2014 16 (Pol. c.)	(0.000)	(0.005)	(0.000)
c_{10}	Distance Elasticity 2014-10 (Ref. c_1)	(0.030^{-1})	$(0.022^{+1.1})$	(0.029^{-1})
		(0.000)	(0.005)	(0.000)
h_1	First Stage - Workers	0 0002***	0 0002***	0 0002***
01	i list stage - workers	(0.0002)	(0.0002)	(0.0002)
h_{2}	First Stage - Managers	0.0011***	0.0011***	0.0011***
02	The stage managers	(0.0011)	(0.000)	(0.000)
μ_1	HQ Manager Productivity 1996-00			-0.089***
				(0.012)
μ_2	HQ Manager Productivity 2001-05 (Rel. μ_1)			0.016
	- • • • • • • • • • • • • • • • • • • •			(0.036)
μ_3	HQ Manager Productivity 2006-10 (Rel. μ_1)			0.061
				(0.043)
μ_4	HQ Manager Productivity 2011-16 (Rel. μ_1)			0.090^{**}
				(0.047)
	Observations	29,104	29,104	29,104

 Table 5: Parameter Estimates

Notes: This table shows the parameter estimates and standard errors from three specifications of the structural model: the standard in counts (Column 1) and efficiency units (Column 2), and incorporating changes in HQ-managerial productivity advantages (Column 3). Exact estimation equations can be found in Section 5 and in Section C.5 of the Online Appendix. Standard errors clustered by establishment's municipality and year in parentheses; *** p < 0.01, ** p < 0.05, * p < 0.1

Figure 3: Model Fit-Changes in Manager-to-Workers Ratio



HQ relative to non-HQ, changes relative to 1996

Both lines from this figure corresponds to the year*HQ fixed effects of a regression on a manager-to-worker ratio at HQ, relative to non-HQ establishments, on establishment, year and year*HQ fixed effects. The solid line corresponds to the ratio observed in the data and presented in Figure 1. The dashed line corresponds to the ratio predicted by the model using the estimated parameters from Table 5.

the most important component with around 51% of the total change.

In terms of spatial wage disparities, increases in the managerial wage gap contributed 43%. Recall that in our model, this change comes from the interplay between two effects. First, a substitution effect: if managers are more expensive, the establishment can substitute managers for workers which can lead to a decrease in the ratio of managers to workers. Second, a HQ-services (firm-scale) effect: as managers become more expensive at HQ, the firm will relocate activities to the satellite establishment, causing an increase in its size and in the demand for HQ-services, thus for managers at the HQ. The two effects combined suggest that an exogenous shock that renders managers more expensive at the HQ location relative to managers at non-HQ locations, can lead to an increase in the relative managerial intensity of the HQ. Finally, changes in the worker wage gap have a negative contribution of -9.1%.

When we perform the decomposition using efficiency units of labor, the managerial wage gap accounts for nearly half of the total change in HQ managerial intensity during this period, while within-establishment wage differences contribute 18%. Allowing HQ manager-enhancing productivity to vary over time (Column 3) yields price difference contributions across and within locations similar to those in Column (2). This suggests that our efficiency-unit calculations partially capture productivity changes over time, as intended. However, the contribution of the distance elasticity rises sharply, while HQ managerial productivity receives a large negative weight. This negative contribution implies that higher managerial productivity at HQ may actually discourage manager concentration in those locations, but increasing firm-level fragmentation offsets this effect since it increases the demand for HQ managers.

	(1)	(2)	(3)
Changes in:	Counts	Eff. Units	Counts
Within Est. Wage Diff - $d \log(v/w)$	15.4%	17.9%	17.5%
Manager Wage Gap - $d \log(v_h/v_i)$	42.7%	49.1%	48.8%
Worker Wage Gap - $d \log(w_h/w_i)$	-9.1%	-9.5%	-10.5%
Distance Elasticity	51.0%	42.5%	107.4%
HQ Manager Productiviy			-63.2%
Total Change	100%	100%	100%

Table 6: Decomposition of the growth of M/L

Notes: This table shows the results of a decomposition of the growth of the manager-to-workers ratio at the HQ relative to non-HQs between 1996 and 2016, into the different components suggested by equation (23). Components were calculated using the parameters from Table 5. Each component was regressed on year and year*HQ fixed effects.

6.4 Number of Establishments

In Section D from the Online Appendix, we present descriptive evidence and numerical simulations to understand how spatial wage differentials, communication costs and agglomeration economies affect the firm's number of establishments and their location. Our estimates suggest that lower relative wages in a municipality offset the negative impact of the distance to the firm's HQ. Thus, a firm might be willing to open an establishment farther from its HQ if it offers a big enough cost advantage. This empirical evidence is supported by our numerical simulations. In particular, we find that changes in wage gaps across locations lead to more firm fragmentation, and this effect strengthens as communication costs fall. In addition, we find that lower communication costs allow firms experiencing moderate levels of agglomeration economies to fragment and open a second establishment where both land and labor are relatively cheap. These results support the predictions of Proposition 3 and are consistent with our empirical findings in Acosta and Lyngemark (2021).

7 Conclusions

Over the last decades, dramatic changes have occurred in the internal spatial organization of firms along two dimensions. First, headquarters (HQ) establishments have become more manager intensive relative to satellite establishments, despite a significant increase in managerial wages at HQ locations. Second, firms have become more fragmented over time, opening more establishments and locating them farther from the firm's HQ. In this paper, we study, theoretically and empirically, how environmental changes faced by firms affect their spatial organization. To study these issues, we first developed a model of a multi-establishment firm to study the effect that changes in relative wages within and across locations and communication costs have on the establishments' labor composition and fragmentation.

Using Danish administrative data, together with commuting-augmented immigration shocks

as the source of identifying variation for changes in the relative supply of workers and managers within and across municipalities, we test the predictions of the model and estimate its structural parameters. We find evidence suggesting that higher wages at HQ locations can lead to more managerial intensive HQ. Moreover, we find that firms are more likely to substitute workers than managers out of their HQ into satellite establishments. Moreover, the across-establishment elasticity of substitution for managers is higher for closer than for distant locations.

Our decomposition indicates that increases in the wage gap of managers between HQ and non-HQ establishments accounts for around 43% of the increase in the managerial intensity at HQ establishments. This can be explained by the increasing demand for manager-intensive HQ services—such as corporate and strategic planning or financial management—which satellite establishments must rely on as they grow larger. The standard labor demand channel (i.e., changes in relative wages within establishments) account for merely 15% for the observed changes. Furthermore, changes in communication costs account for 51%. To the best of our knowledge, this is one of the first papers to specify and estimate a structural model of the internal spatial organization of firms.

Given (i) the increasing importance of ME firms in the aggregate economy, together with (ii) the increasing patterns of firm fragmentation documented in the literature and (iii) the fact that HQs are disproportionally located in large urban areas, these within-firm specialization patterns could significantly affect the aggregate economic geography of a country. Future research should focus on understanding these effects. Future research should also focus on estimating communication and fragmentation costs and the impact they have on firms' spatial decisions.

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Spatial Wage Differentials, Geographic Frictions and the Organization of Labor within Firms

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> Appendix (for online publication)

A Extra Tables and Figures

	SE	Firms	Ν	AE Firms	3
Variable	Mean	Median		Mean	Median
Establishments $(\#)$	1	1		4.08	2
Avg Distance to HQ (Km)	3.55	2.86		89.83	86.56
Distance to HQ (Km)			$\operatorname{non-HQ}$	145.46	115.71
Managers $(\#)$ /Workers $(\#)$	0.22	0.14		0.12	0.08
			HQ	0.16	0.10
			non-HQ	0.19	0.10
Managers $(\#)$	3.16	2		15.83	6
			HQ	10.24	4
			non-HQ	3.87	2
Managers $(HQ/esta)$			non-HQ	11.59	4.67
Executives $(\#)$	1.60	1		2.90	2
			HQ	5.22	2
			non-HQ	1.51	1
Workers (#)	28.32	14		219.11	75
			HQ	113.93	42
			non-HQ	61.04	20
Workers (HQ/esta)			non-HQ	12.23	3.73
Relative Wage (Raw)	1.55	1.40		1.72	1.63
			HQ	1.76	1.67
			non-HQ	1.53	1.42
Relative Wage (Adjusted)	1.00	0.92	HQ	1.09	1.04
			non-HQ	1.03	0.96
$\log(\text{Revenue})$	9.73	9.60		11.58	11.44
$\log(\text{Labor Prod})$	7.04	6.98		7.21	7.11
log(Pop density)	5.86	5.12	HQ	5.95	5.33
			non-HQ	5.50	4.98
Sectoral share (municip)	2.59	1.93	HQ	2.96	2.00
			non-HQ	2.54	1.74

Table A1: Descriptive Statistics

Notes: This table presents the mean and median for some of the variables in our dataset between 1996 and 2016. The number of establishment-year observations are 182,971 for SE firms, 28,641 for HQ in ME firms, and 39,162 for non-HQ in ME firms. The first two columns present the statistics for the SE firms. The third and fourth column present the statistics for ME firms (as a whole) and, where applicable, for HQ and non-HQ establishments separately. Revenue and labor productivity are only available starting in 1999.

	Log(Ho Manager	ourly Wage) Non-Manage
Individual Characteristics	(1)	(2)
Woman	-0 210***	-0 198***
woman	(0.001)	(0.000)
Region of Origin	(0.001)	(0.000)
EU + Nordic	0.069^{***}	0.037^{***}
	(0.004)	(0.001)
Less Developed Europe	-0.084***	0.040***
	(0.009)	(0.006)
Other Europe	0.018^{***}	0.027^{***}
	(0.005)	(0.001)
Amca	-0.027^{++}	$(0.029^{+1.1})$
North America	(0.011) 0.1/3***	(0.001) 0.127***
North America	(0.013)	(0.004)
South and Central America	0.001	0.030***
	(0.015)	(0.002)
Japan	0.112***	0.090***
-	(0.039)	(0.013)
Other Asia	-0.064***	0.035^{***}
	(0.006)	(0.001)
Australia and Oceania	0.143^{***}	0.094^{***}
	(0.027)	(0.005)
Unknown	-0.080	0.045***
	(0.085)	(0.008)
Education [*] (Experience, Tenure)	0 007***	0.006***
Experience	-0.007	(0,000)
Experience ²	0.0002)	-0.000***
Experience	(0.000)	(0.000)
Tenure	0.031***	0.023***
	(0.003)	(0.001)
Tenure ²	-0.001***	-0.001***
	(0.000)	(0.000)
Primary/Secondary	-0.411***	-0.253***
	(0.016)	(0.003)
*Experience	0.034^{***}	0.027^{***}
2	(0.002)	(0.000)
$*Experience^2$	-0.001***	-0.001***
T	(0.000)	(0.000)
*1 enure	-0.004	$-0.010^{-0.01}$
T_{onuro}^2	(0.003)	(0.001)
*1 Chare	(0.000)	(0.000)
Vocational	-0 258***	-0.006**
	(0.016)	(0.003)
*Experience	0.018***	0.005***
4	(0.002)	(0.000)
$*Experience^2$	-0.000***	-0.000***
-	(0.000)	(0.000)
*Tenure	-0.008***	-0.009***
	(0.003)	(0.001)
$*Tenure^2$	0.000	0.000***
	(0.000)	(0.000)
Short/Medium Cycle	-0.215***	-0.087***
,		

Table A2: Wage Regressions

	(0.015)	(0.003)
*Experience	0.26^{***}	0.018^{***}
	(0.002)	(0.000)
$*Experience^2$	-0.001***	-0.000***
	(0.000)	(0.000)
*Tenure	-0.004	-0.006***
	(0.003)	(0.001)
$*Tenure^2$	0.000	0.000^{***}
	(0.000)	(0.000)
Long Cycle	-0.167***	-0.030***
	(0.016)	(0.003)
*Experience	0.32^{***}	0.025^{***}
	(0.002)	(0.000)
$*Experience^2$	-0.001***	-0.000***
	(0.000)	(0.000)
*Tenure	-0.000	-0.003***
	(0.003)	(0.001)
$*Tenure^2$	-0.000	0.000^{***}
	(0.000)	(0.000)
Constant	5.925^{***}	5.490^{***}
	(0.016)	(0.003)
Disco FE	YES	YES
Pstill FE	YES	YES
Establishment*Year FE	YES	YES
Number of Establishments*Year	$294,\!453$	$294,\!453$
Observations	$1,\!104,\!306$	11,772,500
R-squared	0.289	0.331

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Notes: This tables present the estimated coefficients for the regressions that estimate the wage indices. Robust standard errors in parentheses. The omitted categories are Denmark (for region of origin) and unknown education. *** p < 0.01, ** p < 0.05, * p < 0.1

Firm sector	Avg. Executives	Avg. Executives/Workers
Finance and Insurance	2.50	9.23%
IT and information services	1.96	9.02%
Consultancy and legal advice	1.69	8.05%
Research	3.16	8.03%
Textile and leather industry	1.79	7.81%
Advertising and other business services	1.41	7.76%
Electronics industry	3.16	7.31%
Real estate	1.05	7.15%
Chemical industry	3.17	6.83%
Telecommunication	2.33	6.78%
Travel agencies, security, rental services	1.30	6.52%
Manufacturing of electric equipment	2.36	6.52%
Furniture and other industries	1.70	6.38%
Machinery	2.39	6.31%
Wood and paper industry	1.52	6.13%
Medical industry	6.14	5.99%
Plastic, glass and concrete industry	2.02	5.88%
Metal industry	1.44	5.74%
Transport vehicle industry	2.12	5.41%
Transport	1.22	5.25%
Food, beverage and tobacco industry	1.92	3.78%

Table A3: Sectors by Headquarters Services Intensity

Notes: This table shows the average number of executives (our measure of headquarters-services-oriented managers) and their average proportion to the total number of workers among firms in each sector.

	(1)	$\langle 0 \rangle$	(9)
	(1)	(2)	(3)
Dependent variable	$\log\left(\frac{v}{w}\right)_i$	$\log\left(\frac{w_h}{w_i}\right)$	$\log\left(\frac{v_h}{v_i}\right)$
T , ,			
Instrument:	$IS_{Mj}^* - IS_{Lj}^*$	$IS_{Lk}^* - IS_{Lj}^*$	$IS_{Mk}^* - IS_{Mj}^*$
	0.0010***	0.0001***	0.0010***
A: Baseline Estimates	-0.0019	0.0001	0.0010
	(0.0003)	(0.0000)	(0.0001)
Ν	250,774	39,162	39,162
Kleibergen-Paap rk Wald F	30.34	34.61	63.75
B: Shares in 1994	-0.0022***	0.0001^{***}	0.0012***
	(0.0003)	(0.0000)	(0.0002)
Ν	250,774	39,162	39,162
Kleibergen-Paap rk Wald F	42.09	34.85	65.73
C: No Area*Sector Trends	-0.0019***	0.0001^{***}	0.0010***
	(0.0003)	(0.0000)	(0.0001)
Ν	250,774	39,162	39,162
Kleibergen-Paap rk Wald F	30.37	35.02	64.30
0 1			
D: No Big 4 Cities	-0.0048***	0.0002***	0.0011***
2	(0.0016)	(0.0000)	(0.0002)
Ν	180.717	28.504	28.504
Kleibergen-Paan rk Wald F	8 50	63.26	41.85
Kichougen-raap ik walu r	0.09	00.20	41.00

Table A4: First Stage: Establishment-level Wages and Labor Supply Shocks

Notes: This table shows the results of regressions of establishment-level wages on the respective commutingaugmented immigration shock. As dependent variables, column (1) uses the ratio of managers to workers within establishment, column (2) the ratio of workers at the HQ relative to non-HQ, column (3) the ratio of managers at the HQ relative to non-HQ. *j* and *k* correspond to the location of the establishment and its HQ, respectively. Columns (2) and (3) only use non-HQ establishments. Standard errors clustered by establishment's municipality and year. All regressions include establishment and year fixed effects and sector*commuting area pre-trends. Panel A presents the baseline specification (Same as in Table 1). Panel B computes the share-shifter instrument using shares in 1994. Panel C excludes sector*commuting area pre-trends. Panel D excludes all establishments in Copenhagen, Aarhus, Aalborg and Odense. *** p < 0.01, ** p < 0.05, * p < 0.1.

Table A5: Distance to HQ, Copenhagen Dummy and Predicted Immigration

Predicted Imm.	$(1) \\ \log(dist_{ih})$	$(2) \\ \log(dist_{ih})$	$(3) \\ \log(dist_{ih})$	$\begin{pmatrix} (4) \\ CPH_h \end{pmatrix}$	$(5) \\ CPH_h$	$\begin{pmatrix} (6) \\ CPH_h \end{pmatrix}$
$\begin{split} &IS_{Mht}^{F} - IS_{Lht}^{F} \\ &IS_{Lht}^{F} - IS_{Lit}^{F} \\ &IS_{Mht}^{F} - IS_{Mit}^{F} \end{split}$	0.000 (0.007)	-0.000 (0.000)	-0.001 (0.001)	-0.000 (0.000)	0.001^{***} (0.000)	0.002^{***} (0.001)

Notes: This table shows the results from regressions estimating the correlation between the distance between HQ and an establishment, or a dummy if a firm's HQ is in Copenhagen, and the different instrumental variables; i and h denote the location of the establishment i and its HQ h, respectively. All regressions include year, establishment and sector times HQ commuting area fixed effects. The number of observations is 250,774. Standard errors clustered by establishment's municipality and year in parentheses; *** p < 0.01, ** p < 0.05, * p < 0.1

Parameter	Description	Counts
α_1	Labor Share–Cobb-Douglas 1996-00	0.690***
$lpha_2$	Labor Share–Cobb-Douglas 2001-05 (Rel. $\alpha_1)$	(0.056) - 0.318^{***} (0.115)
$lpha_3$	Labor Share–Cobb-Douglas 2006-10 (Rel. $\alpha_1)$	-0.397^{***}
$lpha_4$	Labor Share–Cobb-Douglas 2011-16 (Rel. $\alpha_1)$	(0.111) -0.522^{***} (0.132)
eta_1	Managerial Share–Cobb-Douglas 1996-00	0.190^{***} (0.043)
β_2	Managerial Share–Cobb-Douglas 2001-05 (Rel. $\beta_1)$	0.261^{***}
eta_3	Managerial Share–Cobb-Douglas 2006-10 (Rel. $\beta_1)$	(0.031) 0.334^{***} (0.100)
eta_4	Managerial Share–Cobb-Douglas 2011-16 (Rel. $\beta_1)$	$\begin{array}{c} (0.100) \\ 0.423^{***} \\ (0.107) \end{array}$
c_1	Distance Elasticity 1996-00	0.001
c_2	Distance Elasticity 2001-05 (Rel. c_1)	(0.001) 0.002
c_3	Distance Elasticity 2006-10 (Rel. c_1)	(0.004) 0.020^{***}
c_4	Distance Elasticity 2011-16 (Rel. c_1)	(0.005) 0.039^{***} (0.006)
b_1	First Stage - Workers	0.0002***
b_2	First Stage - Managers	(0.000) 0.0011^{***} (0.000)
	Observations	29,104

Table A6: Time-Varying Parameter Estimates

Notes: Standard errors clustered by establishment's municipality and year in parentheses; table shows the parameter estimates and standard errors from an specification of the structural model where all parameters vary across four time periods: 1996–2000 (reference period), 2001–2005, 2006–2010, and 2011–2016. *** p < 0.01, ** p < 0.05, * p < 0.1

Parameter	Description	Counts
α	Labor Share–Cobb-Douglas	0.647^{***}
2		(0.065)
β	Managerial Share–Cobb-Douglas	0.237***
		(0.053)
c_1	Distance Elasticity 1996-97	0.003***
_	v	(0.001)
c_2	Distance Elasticity 1998-99 (Rel. c_1)	0.001
		(0.003)
c_3	Distance Elasticity 2000-01 (Rel. c_1)	0.001
		(0.004)
c_4	Distance Elasticity 2002-03 (Rel. c_1)	-0.008*
		(0.005)
c_5	Distance Elasticity 2004-05 (Rel. c_1)	-0.008*
		(0.005)
c_6	Distance Elasticity 2006-07 (Rel. c_1)	-0.004
0	Distance Flasticity 2008 00 (Pol. c.)	(0.003)
c_7	Distance Elasticity 2008-09 (Ref. c_1)	(0.014)
Co	Distance Elasticity 2010-11 (Bel. c.)	0.003)
68	Distance Enasticity 2010 11 (Itel. C1)	(0.021)
Co	Distance Elasticity 2012-13 (Rel. c_1)	0.024***
- 3		(0.006)
c_{10}	Distance Elasticity 2014-15 (Rel. c_1)	0.024***
	· · · · · · · · · · · · · · · · · · ·	(0.006)
b_1	First stage parameter	0.0002^{***}
		(0.000)
b_2	First stage parameter	0.0010***
		(0.000)
	Observations	23,664

Table A7: Parameter Estimates - Firms in More than One Labor Market

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Notes: Standard errors clustered by establishment's municipality and year in parentheses; table shows the parameter estimates and standard errors from the baseline specification of the structural model, but including only multi-establishment firms present in more than one labor market, where labor markets are defined by Nielsen (2005). Exact estimation equations can be found in Section 5. *** p < 0.01, ** p < 0.05, * p < 0.1



Figure A1: Changes in the Ratio of Managers to Workers - HQ relative to non-HQ

This figure shows the HQ*year fixed effects from a regression of the log ratio of the establishments' managers to workers on establishment, year and HQ*year fixed effects and commuting area and sector trends, under different specifications. Panel A uses efficiency units of labor. Panel B uses a balanced panel of establishments. Panel C excludes those companies that in years 2010 and 2016 reported to have an establishment outside Denmark, according to the register OFATS. Panel D do not include commuting area and sector trends. Estimates are relative to 1996. Dashed lines correspond to a 95% confidence interval, which include standard errors clustered at the municipality-year level.

Figure A2: Production Structure in a Two-Establishment Firm



This figure shows a representation of the production structure in two-establishment firm. The left circle represent the firm's HQ and the right circle represents the satellite establishment.



Figure A3: Elasticities of Substitution Across Locations

This figure shows the estimated elasticities of substitution of both managers and workers across locations (HQ to non-HQ establishments of ME firms) over distance to the firm's HQ. These estimates come from different regressions of the ratio of workers or managers at the HQ, relative to non-HQ establishments as a function of their respective relative wage, distance to their HQ, and their interaction. Standard errors clustered by establishment's municipality and year. Confidence intervals at the 95% are included.

B Geography of Denmark

Denmark is divided into 98 municipalities. The geographic distribution of population and employment across Danish municipalities is notably uneven. Approximately 36% of the workforce is located in the metropolitan area of Copenhagen, the capital and main city in Denmark. Aarhus, Aalborg and Odense are three medium-size cities that contain 7.1%, 4% and 3.7% of the workforce, respectively. The other half of employment is generated inside the other 94 municipalities, all of which have fewer than 100,000 workers. This distribution can be seen in the left panel of Figure A4, in which we plot the share of employment by municipality in 2016. The four black points in this figure represent the four largest cities, with Copenhagen being the easternmost point.

These geographic disparities are also evident when analyzing some firm organization patterns. In particular, in the right panel of Figure A4 we map the distribution of headquarters establishments across municipalities in 2016. This map shows the high concentration of firms' headquarters around Copenhagen, with Aarhus, Aalborg, Odense and the Triangle Region (*Trekantomraadet*) also displaying a relatively high concentration of headquarters. In particular, the commuting areas of the four main municipalities account for 69% of headquarters of multi-establishment firms. This concentration of HQ establishments is not unique to Denmark. Strauss-Kahn and Vives (2009) show that HQ establishments in the United States are disproportionally located in denser and the largest metropolitan areas. Figure A5 shows the 25 commuting areas in Denmark, as defined by Nielsen (2005) based on commuting flows across municipalities from 2004. Each commuting area has on average 4 municipalities.



Figure A4: Economic Geography of Denmark

The left panel shows the distribution of workers across municipalities. The right panel shows the distribution of headquarters across municipalities. Black dots represent the four main cities, with Copenhagen being the easternmost point.





This figure shows the commuting areas as defined by Nielsen (2005) based on commuting flows across municipalities in 2004. The black dots represent the four main cities, with Copenhagen being the easternmost point.

C Mathematical Appendix

C.1 Model Derivations

In this appendix, we present the main derivations from our model. Recall that the model starts from the following production function:

$$Y_{ij} = A_i L_i^{\alpha} \left[\min\{\lambda \mu_j m_i, \theta \tau_j H\} \right]^{\beta},$$

where $\alpha, \beta \in (0,1)$ and $\alpha + \beta < 1$, and $\mu_o > 1$ and $\mu_s = 1$, and $\lambda > 0$ and $\theta > 0$ are technology-determined constants.

Centralized Solution

Define the total number of managers in the establishment as $M_1 = m_1 + m_h$. With a singleestablishment firm, and given that the HQ's location is fixed at location o, the demand for both types of workers and the firm's total profits is given by:

$$M_{1}^{I} = \left[\alpha^{\alpha} \beta^{1-\alpha} \left(\frac{\theta \lambda}{\theta + \lambda} \right)^{\beta} \mu^{\beta} w_{o}^{-\alpha} v_{o}^{\alpha-1} p A_{i} \right]^{\frac{1}{\kappa}},$$

$$L_{1}^{I} = \left[\alpha^{1-\beta} \beta^{\beta} \left(\frac{\theta \lambda}{\theta + \lambda} \right)^{\beta} \mu^{\beta} w_{o}^{\beta-1} v_{o}^{-\beta} p A_{i} \right]^{\frac{1}{\kappa}},$$

$$\Pi_{o}^{I} = \kappa \left[\alpha^{\alpha} \beta^{\beta} \left(\frac{\theta \lambda}{\theta + \lambda} \right)^{\beta} w_{o}^{-\alpha} \left(\frac{v_{o}}{\mu} \right)^{-\beta} p A_{i} \right]^{\frac{1}{\kappa}} - r_{o},$$
(24)

where the superscript I indicates that the equations correspond to the solution of a single establishment firm, and $\kappa = 1 - \alpha - \beta$. An interior solution exists as long as $\alpha + \beta \in (0, 1)$. This solution shows us that single-establishment firms hire more of both types of workers when there are higher agglomeration economies, lower input prices, or a higher price for the final good, which can be thought of as a proxy for the demand faced by the firm. Moreover, more productive firms hire more workers and are more profitable, as expected.

Fragmented Solution

Define the total number of managers in each establishment as $M_1 = m_1 + m_h$ and $M_2 = m_2$. When the firm has two establishments, with HQ in o and satellite establishment in $j \in \{o, s\}$, the solution of the firm's problem in equation (6) results in the following demand for both types of workers at each establishment and the firm's total profits:

$$M_{1o}^{II} = \left(\alpha^{\alpha}\beta^{1-\alpha}pA_{i}\right)^{\frac{1}{\kappa}} \left\{ \left(\lambda^{\beta}\mu^{\beta}w_{o}^{-\alpha}v_{o}^{\alpha-1}\right)^{\frac{1}{\kappa}} + \frac{1}{\theta\tau_{j}\mu} \left[\frac{w_{j}^{-\alpha}}{\left(\frac{v_{j}}{\lambda\mu_{j}} + \frac{v_{o}}{\theta\tau_{j}\mu}\right)^{1-\alpha}}\right]^{\frac{1}{\kappa}} \right\}, \quad (25)$$

$$L_{iI}^{II} = \left(\alpha^{1-\beta}\beta^{\beta}w^{\beta-1}\left(\frac{v_{o}}{\lambda\mu_{j}}\right)^{-\beta}pA_{i}\right)^{\frac{1}{\kappa}} \quad (26)$$

$$L_{1o}^{II} = \left(\alpha^{1-\beta}\beta^{\beta}w_{o}^{\beta-1}\left(\frac{v_{o}}{\lambda\mu}\right)^{-\beta}pA_{i}\right)^{\kappa}, \qquad (26)$$

$$M_{2j}^{II} = \frac{1}{\lambda\mu_j} \left[\alpha^{\alpha}\beta^{1-\alpha}w_j^{-\alpha} \left(\frac{v_j}{\lambda\mu_j} + \frac{v_o}{\theta\tau_j\mu}\right)^{\alpha-1} pA_i \right]^{\frac{1}{\kappa}},$$
(27)

$$L_{2j}^{II} = \left(\alpha^{1-\beta}\beta^{\beta}w_{j}^{\beta-1}\left(\frac{v_{j}}{\lambda\mu_{j}} + \frac{v_{o}}{\theta\tau_{j}\mu}\right)^{-\beta}pA_{i}\right)^{\frac{1}{\kappa}},$$
(28)

$$\Pi_{(o,j)}^{II} = (\tilde{\kappa}pA_i)^{\frac{1}{\kappa}} \left\{ \left[w_o^{-\alpha} \left(\frac{v_o}{\lambda\mu} \right)^{-\beta} \right]^{\frac{1}{\kappa}} + \left[w_j^{-\alpha} \left(\frac{v_j\tau_j}{\lambda\mu_j} + \frac{v_o}{\theta\mu} \right)^{-\beta} \tau_j^{\beta} \right]^{\frac{1}{\kappa}} \right\} - r_o - r_j, \quad (29)$$

where the superscript II indicates that the equations correspond to the solution of a twoestablishment firm; $\Pi_{o,j}^{II}$ denotes the profits of a firm with two establishments located in o and $j \in \{o, s\}$, respectively; $\kappa = 1 - \alpha - \beta$ and $\tilde{\kappa} = \kappa \alpha^{\alpha/\kappa} \beta^{\beta/\kappa}$. Several things can be seen here. First, a reduction in communication costs $(\uparrow \tau)$ leads to a increase in the demand for both types of workers in the satellite establishment, as well as an increase in the firm's profits. Second, higher wages lead to a decrease in factor demands and in the profitability of the firm. Land prices also lead to a decrease in the firm's profit. Third, demand for both types of workers in the satellite establishment depends not only on the price of workers and managers in that establishments, but also on the price of HQ managers. This is because production of the final good at any establishment requires HQ services, which is produced by headquarters-type managers. Moreover, notice that an increase in the price of HQ managers leads to a decrease in the demand for both managers and workers at both establishments. Fourth, stronger decreasing returns to scale lead to lower profits and lower factor demand.

Number of Establishments and their Locations

In the first stage of the firm's problem, it has to choose the number of establishments and their respective locations. Given the model's current setting, this decision is equivalent to choosing between having only one establishment, opening a second in o, or opening a second in s. The solution to this problem comes from comparing equations (24) and (29). For instance, a firm will choose to have two establishments, with the HQ located in o and the second establishment located in $j \in \{o, s\}$, if $\Pi_{(o,j)}^{II} > \Pi_{o}^{I}$ and $\Pi_{(o,j)}^{II} > \Pi_{(o,k)}^{II}$, for $k \neq j, k \in \{o, s\}$.

In particular, a firm will open its satellite establishment in s if the marginal benefit of having that establishment is larger than the extra fixed cost of opening it. This marginal benefit depends on a trade-off between wage differences, communication costs and agglomeration advantages. On the one hand, wage differences between a firm's HQ and other locations create an incentive for the firm to open a new establishment in places where wages are relatively low. On the other hand, by opening new establishments, firms incur extra communications costs and could miss out on the productivity advantages present at the HQ location. Deeper comparative statics are explored below.

We can extend our model to consider the case where the firm can open a third establishment. To keep the model's development tractable, we fix the number of possible locations to (o, s), but allow for one HQ–located in o–and two satellite establishments located in $j \in \{o, s\}$ and $k \in \{o, s\}$, respectively. In this case, the solution to this problem yields a set of equations highly similar to equations (25) to (29) but now includes a term containing the price of managers in the third establishment (when this is active and adjusted by communication costs and productivity). In this case, the expression for the total number of managers at the HQ and the total profits for a three-establishment firm are:

$$M_{1o}^{III} = \left(\alpha^{\alpha}\beta^{1-\alpha}pA_{i}\right)^{\frac{1}{\kappa}} \left\{ \left(\lambda^{\beta}\mu^{\beta}w_{o}^{-\alpha}v_{o}^{\alpha-1}\right)^{\frac{1}{\kappa}} + \frac{1}{\theta\mu} \left[\frac{\left[\left(\tau_{j}^{\beta}w_{j}^{-\alpha}\right)^{\frac{1}{1-\alpha}} + \left(\tau_{k}^{\beta}w_{k}^{-\alpha}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}}{\left(\frac{v_{o}}{\theta\mu} + \frac{v_{j}\tau_{j}}{\lambda\mu_{j}} + \frac{v_{k}\tau_{k}}{\lambda\mu_{k}}\right)^{1-\alpha}} \right]^{\frac{1}{\kappa}} \right\},$$
(30)

$$\Pi_{(o,j,k)}^{III} = (\tilde{\kappa}pA_i)^{\frac{1}{\kappa}} \left\{ \left[w_o^{-\alpha} \left(\frac{v_o}{\lambda\mu} \right)^{-\beta} \right]^{\frac{1}{\kappa}} + \left[\left[\left(\tau_j^{\beta} w_j^{-\alpha} \right)^{\frac{1}{1-\alpha}} + \left(\tau_k^{\beta} w_k^{-\alpha} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} \left(\frac{v_o}{\theta\mu} + \frac{v_j\tau_j}{\lambda\mu_j} + \frac{v_k\tau_k}{\lambda\mu_k} \right)^{-\beta} \right]^{\frac{1}{\kappa}} \right\} - r_o - r_j - r_k.$$

$$(31)$$

It can be observed that the intuition described for the main results above also hold for this particular case.

C.2 Model Predictions and Main Theoretical Results

In this subsection, we present the main results of our model. In particular, we show how the firm's relative labor demands across and within establishments change when wages and communication costs change. For ease of presentation, let us assume that the firm locates its second establishment in j = s.

Labor composition within Establishments

Start by analyzing the solution of the labor demand problem of a single-establishment firm from equation (24). The ratio of manager to workers for this firm is given by

$$\left(\frac{M_1}{L_1}\right)_{SE} = \frac{\beta}{\alpha} \frac{w_o}{v_o}.$$
(32)

Notice that this ratio depends only on the establishment's relative wage between managers and workers. Therefore, when a single-establishment firm faces an exogenous shock that makes managers more expensive relative to workers, it can only respond by hiring fewer managers and/or more workers. This is not the case for establishments that belong to a multi-establishment firm. First, consider the ratio of managers to workers at non-HQ establishments given by equations (27) and (28):

$$\left(\frac{M_2}{L_2}\right)_{ME} = \frac{\beta}{\alpha} \frac{w_s}{v_s} \left(1 + \frac{v_o}{v_s} \frac{\lambda}{\theta \tau \mu}\right)^{-1}.$$
(33)

Just as single-establishment firms, this ratio depends negatively on the relative wages between managers and workers within the establishment. However, notice that it also depends negatively on the price of managers at the HQ relative to the establishment. This implies that an exogenous increase in the price of HQ managers raises the cost of the managerial inputs used at the non-HQ establishment, decreasing the demand for local managers. Finally, note that lower communication costs (τ) and higher HQ agglomeration economies magnify this effect.

Now, consider the ratio of manager to workers at the HQ given by equations (25) and (26):

$$\left(\frac{M_1}{L_1}\right)_{ME} = \frac{\beta}{\alpha} \frac{w_o}{v_o} \left\{ 1 + \left[\left(\frac{\theta\tau}{\lambda}\right)^{\beta} \left(\frac{w_o}{w_s}\right)^{\alpha} \left(\frac{v_s}{v_o}\frac{\theta\tau\mu}{\lambda} + 1\right)^{\alpha-1} \right]^{\frac{1}{1-\alpha-\beta}} \right\}.$$
 (34)

In addition to the standard labor demand channel, this ratio also depends on the wage gap of workers and managers between both locations, communication costs and agglomeration economies. This equation suggests that lower wages at a non-HQ establishment would lead to a higher manager-to-workers ratio at at the HQ. Moreover, lower communication costs magnify the effect that differences in the wages of managers and workers have on the HQ's managerial intensity. Finally, lower communication costs themselves have an ambiguous effect on managerial intensity at the HQ. On one hand, since non-HQ establishments receive more H, there will be an incentive to produce more HQ services and hire more managers at the HQ. On the other hand, if the establishment receives a higher quantity of HQ services, it also needs to hire more local managers, which would drive down total profits and the demand for H.

It is evident now that when facing an exogenous wage shock, multi-establishment firms have an extra margin of adjustment that single-establishment firms do not have: They can substitute labor across locations. To see this more clearly, consider first an exogenous shock that renders workers (non-managers) at the central location more expensive. Given this shock, a multiestablishment firm has two options: It can either substitute workers for managers within the establishment or it can substitute workers from the HQ to the non-HQ establishment. Either of these responses would cause a decrease in the number of workers at the HQ (L_o), leading to an increase in HQ managerial intensity.

Consider now an exogenous shock that renders managers in the central location more expensive. Given that HQ managers are used in the production of both the final good and HQ services (which are used by all establishments), the direction of this effect is not as straightforward as the previous one. Using a Slutsky-type decomposition, the total effect of an exogenous increase in the wage of managers at the HQ location on the manager-to-worker ratio at the HQ can be decomposed as the sum of four effects:

$$\frac{\partial M_1/L_1}{\partial v_o} = \frac{\partial (M_1/L_1)^c}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial v_o}$$

$$\frac{\partial M_1/L_1}{\partial v_o} = \underbrace{\frac{\partial (M_1/L_1)^c}{\partial (v_o/w_o)} \cdot \frac{\partial (v_o/w_o)}{\partial v_o}}_{\text{Standard Subst. Effect}} + \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/w_o)} \cdot \frac{\partial (v_o/w_o)}{\partial v_o}}_{\text{Standard Scale Effect}} + \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o}}_{\text{Standard Scale Effect}} + \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o}}_{\text{Standard Scale Effect}} + \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o}}_{\frac{\partial V_0}{\partial v_o}},$$

$$(35)$$

HQ Services / Firm Scale Effect

where the superscript c denotes the conditional labor demand functions that result from the firm's cost minimization problem, and Y_1 and Y_2 correspond to the total amount of output produced at HQ and non-HQ establishments, respectively. In Appendix C.3, we show the details of this decomposition.

The first two terms on the right-hand side are the standard substitution and scale effects from a regular Slutsky-type equation. On one hand, if managers are more expensive, the establishment can substitute managers for workers (substitution effect). On the other hand, since the production of the final output requires both managers and workers, more expensive managers lead to a reduction in the establishment's output, which drives down the number of both types of workers at the HQ (scale effect). Both of these effects lead to a decrease in the ratio of managers to workers.

However, keeping constant the wage differences between managers and workers within the establishment, other effect arises. As HQ managers become more expensive, the firm will relocate both managers and workers to the satellite establishment. As this establishment becomes

larger, the demand for HQ-services will also increase, hence increasing the demand for managers at the HQ and the ratio of managers to workers.

These two forces change the ratio of manager to workers at the HQ in different directions. However, it can be shown that the latter effect dominates the former. Therefore, an exogenous shock that renders the price of managers more expensive at the central location can lead to an increase in the HQ's managerial intensity, keeping fixed within-establishment wage differences. Note that, taking together the effects of an increase in the wage ratio v_o/v_s on the manager-toworkers ratio at HQ (equation 34) and non-HQ establishments (equation 33), we can conclude that a wider managerial wage gap leads to a more manager-intensive HQ relative to non-HQ establishments.

We formalize all these results in the next proposition. Formal proof of all propositions are included in a Mathematical Appendix.

Proposition 1. Managerial Intensity

(a) Higher relative wages at the establishment's location $(\uparrow \frac{v}{w})$ lead to less manager-intensive establishments $(\downarrow \frac{M}{L})$. This holds for every establishment in the economy.

(b) For multi-establishment firms, a wider worker wage gap across locations $(\uparrow \frac{w_o}{w_s})$ leads to a more manager-intensive HQ relative to the non-HQ establishment $(\uparrow \frac{M_h/L_h}{M_i/L_i})$.

(c) For multi-establishment firms, a wider managerial wage gap across locations $(\uparrow \frac{v_o}{v_s})$ leads to a more manager-intensive HQ relative to the non-HQ establishment $(\uparrow \frac{M_h/L_h}{M_i/L_i})$. Lower communication costs $(\uparrow \tau)$ magnify the effect.

Proof. (a) First, for single-establishment firms the proof is straightforward as:

$$\frac{\partial (M_1/L_1)}{\partial (v_o/w_o)} = -\frac{\beta}{\alpha} \left(\frac{w_o}{v_o}\right)^2 < 0$$

For non-HQ establishments belonging to multi-establishment firms, from equation (33) take the partial derivative of the manager to worker ratio with respect to the respective wage ratio (v_s/w_s) :

$$\frac{\partial (M_2/L_2)}{\partial (v_s/w_s)} = -\frac{\beta}{\alpha} \left(\frac{w_s}{v_s}\right)^2 \left(1 + \frac{v_o}{v_s}\frac{\lambda}{\theta\tau\mu}\right)^{-1} < 0$$

For HQ establishments of multi-establishment firms, from equation (34) take the partial derivative of the manager to worker ratio with respect to the respective wage ratio (v_o/w_o) :

$$\frac{\partial (M_1/L_1)}{\partial (v_o/w_o)} = -\frac{\beta}{\alpha} \left(\frac{w_o}{v_o}\right)^2 \left\{ 1 + \left[\left(\frac{\theta\tau}{\lambda}\right)^\beta \left(\frac{w_o}{w_s}\right)^\alpha \left(\frac{v_s}{v_o}\frac{\theta\tau\mu}{\lambda} + 1\right)^{\alpha-1} \right]^{\frac{1}{1-\alpha-\beta}} \right\} < 0$$

(b) Start by finding the ratio between equations (34) and (33):

$$\frac{(M_1/L_1)}{(M_2/L_2)} = \frac{(v_s/w_s)}{(v_o/w_o)} \left\{ 1 + \left[\left(\frac{\theta\tau}{\lambda}\right)^{\beta} \left(\frac{w_o}{w_s}\right)^{\alpha} \left(\frac{v_s}{v_o}\frac{\theta\tau\mu}{\lambda} + 1\right)^{\alpha-1} \right]^{\frac{1}{1-\alpha-\beta}} \right\} \left(1 + \frac{v_o}{v_s}\frac{\lambda}{\theta\tau\mu} \right)$$

The partial derivative of this ratio with respect to w_o/w_s is proportional to:

$$\frac{\partial \frac{(M_1/L_1)}{(M_2/L_2)}}{\partial w_o/w_s} \propto \frac{1-\beta}{1-\alpha-\beta} \left[\left(\frac{\theta\tau}{\lambda}\right)^\beta \left(\frac{w_o}{w_s}\right)^\alpha \left(\frac{v_s}{v_o}\frac{\theta\tau\mu}{\lambda}+1\right)^{\alpha-1} \right]^{\frac{1}{1-\alpha-\beta}} > 0$$

(c) On one hand, from equation (34) it can be observed that:

$$\frac{\partial (M_1/L_1)}{\partial (v_o/v_s)} > 0$$

On the other hand, from equation (33) it can be observed that:

$$\frac{\partial (M_2/L_2)}{\partial (v_o/v_s)} < 0$$

Therefore, it must be that:

$$\frac{\partial \frac{(M_1/L_1)}{(M_2/L_2)}}{\partial v_o/v_s} > 0$$

We can derive the elasticities of the HQ manager-to-worker ratio with respect to a change in the wage gap of either workers or managers across locations:

Corollary 1. Keeping the within-establishment wage ratio $\left(\frac{v}{w}\right)$ constant and defining $M_1 = m_1 + m_h$, the elasticities of the HQ ratio of managers to workers, with respect to changes in the wage gap of workers and managers across locations are given by:

(a) For workers:
$$\frac{\partial \log(M_1/L_1)}{\partial \log(w_o/w_s)} = \frac{\alpha}{1-\alpha-\beta} \cdot (1-\omega) > 0.$$

(b) For managers: $\frac{\partial \log(M_1/L_1)}{\partial \log(v_o/v_s)} = \frac{1-\alpha}{1-\alpha-\beta} \cdot (1-\gamma) \cdot (1-\omega) > 0.$

where $\gamma = \frac{v_o/\theta\tau\mu}{(v_s/\lambda) + (v_o/\theta\tau\mu)} > 0$ corresponds to the cost share of non-HQ managers on the total managerial costs of the non-HQ establishment, and $\omega = \frac{v_o m_1}{v_o M_1} > 0$ corresponds to the cost share of production managers on total managerial costs at the HQ.

Labor composition across Establishments

From our model, we can also write two equations that can help us understand the substitution patterns of workers and managers across establishments. Empirically, these two equations are useful for two reasons. First, since these equations will yield relatively simple linear regressions, their reduced-form estimates will help us test some of the model's predictions and, therefore, its validity. Second, the across-location elasticities of substitution implied by these equations provide important variation to identify the structural parameters of our model.

Under our current assumptions, we can write the ratio of workers at the HQ relative to the non-HQ establishment as

$$\frac{L_1}{L_2} = \left[\left(\frac{w_o}{w_s} \right)^{\beta-1} \left(\frac{v_s}{v_o} \mu + \frac{\lambda}{\theta \tau} \right)^{\beta} \right]^{\frac{1}{1-\alpha-\beta}}.$$
(36)

Similarly, for managers:

$$\frac{M_1}{M_2} = \frac{\lambda}{\theta \tau \mu} + \frac{1}{\mu} \left[\left(\frac{w_o}{w_s} \right)^{-\alpha} \left(\frac{v_s}{v_o} \mu + \frac{\lambda}{\theta \tau} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha-\beta}}.$$
(37)

From these expressions, we highlight three effects. First, lower communication costs lead to a decrease in the ratios of workers and managers at the HQ relative to the other establishment. This is caused by the fact that with lower communication costs, non-HQ establishments receive more services from the HQ and hire more managers and workers to produce more output. Second, higher HQ manager-specific productivity (μ) has a positive effect on the ratio of workers at the HQ relative to the non-HQ, but an ambiguous effect on the managerial ratio. On one hand, since managers are now more productive at the HQ, there is an incentive to increase the number of managers at the HQ relative to the other establishment. However, this rise in productivity also leads to a higher production of HQ services, thus driving up the demand for local managers at the non-HQ establishment.

Third, the share of workers and managers at the HQ depends negatively on the respective wage gap across locations. This result suggests that the size of the HQ relative to the non-HQ establishment, decreases when the wage gap across locations is wider. Even though this is an expected result, it is worth discussing the across-location elasticities of substitution, since this variation will help us identify the model parameters in the estimation from Section 6.2. From the two equations above, we calculate the respective across-location elasticities of substitution of workers and managers as

$$\frac{\partial \log(L_1/L_2)}{\partial \log(w_o/w_s)} = \frac{\beta - 1}{1 - \alpha - \beta} < 0,$$

$$\frac{\partial \log(M_1/M_2)}{\partial \log(v_o/v_s)} = \frac{\alpha - 1}{1 - \alpha - \beta} \cdot \frac{v_s/\lambda}{(v_s/\lambda) + (v_o/\theta\mu\tau)} \cdot \frac{v_o m_1}{v_o M_1} < 0,$$
(38)

where $M_1 = m_1 + m_h$. While the elasticity of substitution of workers across establishments is constant, this is not the case for managers. The latter elasticity depends on two cost shares: (i) the cost share of non-HQ managers on the total managerial cost of the non-HQ establishment, and (ii) the cost share of production managers on the total managerial costs at the HQ. In particular, if production is not dependent on HQ services, the elasticity converges to a constant given by $\alpha - 1/1 - \alpha - \beta$. Moreover, as the use of HQ services increases, this elasticity converges to zero. Therefore, this elasticity implies that when firms are very dependent on the within-firm public good, we would observe a low degree of substitution of managers across establishments for a given exogenous change in the wage gap of managers. Furthermore, if $\beta < \alpha$, the acrosslocation elasticity of substitution for workers will always be more negative than the one for managers. Finally, consider the interactions between the previous elasticities and communication costs. From equation (38), we observe that lower communication costs make this elasticity more negative, thus magnifying an increase in the managerial wage gap.²⁴ These results can be summarized as:

Proposition 2. Substitution Across Locations

(a) A steeper wage gap for workers $(\uparrow \frac{w_o}{w_s})$ leads to a lower share of workers at the HQ $(\downarrow \frac{L_h}{L_i})$. (b) A steeper wage gap for managers $(\uparrow \frac{v_o}{v_s})$ leads to a lower share of managers at the HQ

²⁴This is a result of the assumed complementarity between local managers and HQ services. In Appendix C.4, we show that the direction of this cross-derivative is zero if the managerial bundle is we assume a Cobb-Douglas production function for the managerial bundle.

 $(\downarrow \frac{M_h}{M_i})$. Lower communication costs $(\uparrow \tau)$ magnify the effects.

Proof. (a) Consider equation (36). Take logs and its partial derivative with respect to $\log(w_o/w_s)$:

$$\frac{\partial \log(L_1/L_2)}{\partial \log(w_o/w_s)} = (\beta - 1) < 0$$

(b) Consider equation (37). That equation is proportional to:

$$\frac{M_1}{M_2} \propto \frac{1}{\mu} \left[\left(\frac{w_o}{w_s} \right)^{-\alpha} \left(\frac{v_s}{v_o} \mu + \frac{\lambda}{\theta \tau} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha-\beta}}$$

Taking logs of both sides of the equations and its partial derivative with respect to $\log(v_o/v_s)$:

$$\frac{\partial \log(M_1/M_2)}{\partial \log(v_o/v_s)} \propto -\frac{1-\alpha}{1-\alpha-\beta} \cdot \frac{1}{\frac{v_o}{v_s} + \frac{\lambda}{\theta\tau\mu} \left(\frac{v_o}{v_s}\right)^2} < 0$$

Moreover, notice that a larger τ makes the previous expression even more negative.

Number of establishments

All of the comparative statics so far have assumed that the firm has two establishments, located in o and s. Nevertheless, we would like to know how the firm's decision on whether to have a second establishment, and its eventual location, changes with wages, communication costs and productivity. Recall that these decisions involve fixed costs, which in our model are connected to the price of local land. Using equations (24) and (29), our model gives the following prediction regarding the change in the firm's number of establishments:

Proposition 3. Firms are more likely to open a satellite establishment with a decrease in communication costs, or an increase in the price of inputs at the HQ (for either type of labor or land), productivity advantages at the HQ, or firm-specific productivity.

Proof. Taking the difference between equations (29) and (24) yields the following expression:

$$\Delta \Pi = \Pi_{(o,s)}^{II} - \Pi_{o}^{I}$$
$$= (\tilde{\kappa}pA)^{\frac{1}{\kappa}} \left\{ \left[w_{o}^{-\alpha} \left(\frac{v_{o}}{\lambda \mu} \right)^{-\beta} \right]^{\frac{1}{\kappa}} \left[1 - \left(\frac{\theta}{\theta + \lambda} \right)^{\frac{\beta}{\kappa}} \right] + \left[w_{s}^{-\alpha} \left(\frac{v_{s}\tau}{\lambda} + \frac{v_{o}}{\theta \mu} \right)^{-\beta} \tau^{\beta} \right]^{\frac{1}{\kappa}} \right\} - r_{s}$$

Rewrite this expression as:

$$\Delta \Pi = \Phi_1 + \Phi_2 - r_s,$$

where Φ_1 and Φ_2 are given by their respective expressions in the equation above. Proceeding with the comparative statics:

(a) Communication costs (τ) : notice that:

$$\frac{\partial \Delta \Pi}{\partial \tau} = \frac{\partial \Phi_2}{\partial \tau} = \Phi_2 \frac{\beta}{\kappa \tau} \frac{\frac{\upsilon_o}{\theta \mu}}{\frac{\upsilon_s \tau}{\lambda} + \frac{\upsilon_o}{\theta \mu}} > 0$$

(b) Agglomeration economies in the central location (μ) :

$$\frac{\partial \Delta \Pi}{\partial \mu} = \frac{\partial \Phi_1}{\partial \mu} + \frac{\partial \Phi_2}{\partial \mu} = \Phi_1 \frac{\beta}{\kappa \mu} + \Phi_2 \frac{\beta}{\kappa \mu} \frac{\frac{v_o}{\partial \mu}}{\frac{v_o \tau}{\lambda} + \frac{v_o}{\partial \mu}} > 0$$

(c) Higher firm-specific productivity (A) and higher differences in the price of land (r_s) : for both of these variables it is clear that: $\frac{\partial \Delta \Pi}{\partial A} > 0$ and $\frac{\partial \Delta \Pi}{\partial r_s} < 0$.

(d) For both the wage gaps for workers (w_o/w_s) and the wage gaps for managers (v_o/v_s) , a simple analytic expression cannot be found. Nonetheless, the numeric simulations from Section 6.4 show that higher wage gaps for either type of workers leads to an increase in the average number of establishments.

C.3 Slutsky Decomposition of the HQ Managerial Intensity

Consider the cost minimization problem of the two-establishment firm:

$$\min_{L_1, L_2, m_1, m_2, m_h} w_o L_1 + w_s L_2 + v_o m_1 + v_s m_2 + v_o m_h$$
s.t. $\bar{Y}_1 = L_1^{\alpha} [\min\{m_1, m_h\}]^{\beta}$
 $\bar{Y}_2 = L_2^{\alpha} [\min\{m_2, \tau m_h\}]^{\beta},$

where we assumed $\lambda = \theta = 1$ and $\mu_o = \mu_s = 1$ without loss of generality. The solution of this problem yields the conditional labor demand for workers and managers at both locations:

$$L_{1}^{c} = \left[\bar{Y}_{1}\left(\frac{\beta}{\alpha}\frac{w_{o}}{v_{o}}\right)^{-\beta}\right]^{\frac{1}{\alpha+\beta}}, \qquad (39)$$

$$L_{2}^{c} = \left[\frac{Y_{2}}{\tau^{\beta}}\left(\frac{\beta}{\alpha}\frac{w_{s}}{v_{o}}\right)^{-\beta}\right]^{\frac{1}{\alpha+\beta}}, \qquad (39)$$

$$M_{1}^{c} = \left(\frac{\beta}{\alpha v_{o}}\right)^{\frac{\alpha}{\alpha+\beta}}\left[\left(\bar{Y}_{1}w_{o}^{\alpha}\right)^{\frac{1}{\alpha+\beta}} + \left(\bar{Y}_{2}\tau^{-\beta}w_{s}^{\alpha}\right)^{\frac{1}{\alpha+\beta}}\right], \qquad (40)$$

$$M_{2}^{c} = \left[\frac{Y_{2}}{\tau^{-\alpha}}\left(\frac{\beta}{\alpha}\frac{w_{s}}{v_{o}}\right)^{\alpha}\right]^{\frac{1}{\alpha+\beta}}, \qquad (40)$$

where $M_1^c = m_1^c = m_h^c$ and $M_2^c = m_2^c$. From equations (40) and (39), we can write the conditional relative demand at the HQ as:

$$\left(\frac{M_1}{L_1}\right)^c = \frac{\beta}{\alpha} \frac{w_o}{v_o} + \frac{\beta}{\alpha} \frac{w_o}{v_o} \left(\frac{\bar{Y}_2}{\bar{Y}_1} \tau^{-\beta} \left(\frac{w_s}{w_o}\right)^{\alpha}\right)^{\frac{1}{\alpha+\beta}}.$$
(41)

From the profit maximization problem in the text, we can find and write the optimal level of output in both establishments as:

$$Y_1 = \left(\alpha^{\alpha}\beta^{\beta}p^{\alpha+\beta}\left(\frac{v_o}{w_o}\right)^{-\alpha}\left(\frac{v_o}{v_s}\right)^{-\alpha-\beta}v_s^{-\alpha-\beta}\right)^{\frac{1}{1-\alpha-\beta}},\tag{42}$$

$$Y_2 = \left(\alpha^{\alpha}\beta^{\beta}p^{\alpha+\beta}w_s^{-\alpha}\left(1+\frac{v_o}{v_s\tau}\right)^{-\beta}v_s^{-\beta}v_o^{-\beta}\right)^{\frac{1}{1-\alpha-\beta}}.$$
(43)

With all these elements, consider the Slutsky decomposition –equation (35)– from the text:

$$\begin{split} \frac{\partial M_1/L_1}{\partial v_o} &= \frac{\partial (M_1/L_1)^c}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial v_o} \\ &= \underbrace{\frac{\partial (M_1/L_1)^c}{\partial (v_o/w_o)} \cdot \frac{\partial (v_o/w_o)}{\partial v_o}}_{A \equiv \text{Standard Subst. Effect}} + \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/w_o)} \cdot \frac{\partial (v_o/w_o)}{\partial v_o}}_{B \equiv \text{Standard Scale Effect}} \\ &+ \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o}}_{C \equiv \text{Firm Scale Effect}} \end{split}$$

Terms A, B and C can be written as:

$$A = \frac{-\beta}{\alpha} \frac{w_o}{v_o^2} \left[1 + \left(\frac{\bar{Y}_2}{\bar{Y}_1} \tau^{-\beta} \left(\frac{w_s}{w_o} \right)^{\alpha} \right)^{\frac{1}{\alpha+\beta}} \right] < 0,$$

$$B = \frac{-\beta}{(\alpha+\beta)(1-\alpha-\beta)} \frac{w_o}{v_o v_s} \left(\frac{\bar{Y}_2}{\bar{Y}_1} \tau^{-\beta} \left(\frac{w_s}{w_o} \right)^{\alpha} \right)^{\frac{1}{\alpha+\beta}} < 0,$$

$$C = \frac{\beta}{\alpha(\alpha+\beta)(1-\alpha-\beta)} \frac{w_o}{v_o^2} \left(\frac{\bar{Y}_2}{\bar{Y}_1} \tau^{-\beta} \left(\frac{w_s}{w_o} \right)^{\alpha} \right)^{\frac{1}{\alpha+\beta}} \underbrace{\left[-\beta \frac{v_o/\tau}{v_s + v_o/\tau} + \frac{(\alpha+\beta)}{\uparrow \text{Scale HQ}} \right] > 0. \quad (44)$$

Notice that term A corresponds to the direct substitution effect between managers and workers given by an increase in the wage of managers. Term B corresponds to the within-establishment scale or output effect. That is, the increase in the wage of managers leads to an increase in the cost of production, causing less of the good to be sold, further affecting the demand for both types of labor. Finally, term C comes from changes in the wage of managers at the HQ, relative to non-HQ locations, keeping the within-establishment wage ratios fixed. This effect encompasses two scale effects that arise due to the fact that all of the firm's establishments need HQ services in order to produce final output. On one hand, that an increase in the price of HQ managers make production at the non-HQ establishment more expensive since it increases the cost of the managerial bundle. This effect will drive down the demand for local and headquarter managers, driving down M_1/L_1 . On the other hand, since non-HQ establishments are now cheaper, the firm wants to make these establishments larger. The increase in the size of the satellite establishments, generates an increase in the demand for HQ services, which are an input necessary for production. Since HQ services are a manager intensive good, this would cause and increase in the demand for managers at the headquarters. It can be seen easily that the latter effect dominates the former.

C.4 Model with a Cobb-Douglas Managerial Bundle

In this appendix, we derive our main theoretical results assuming the following Cobb-Douglas managerial bundle:

$$Y_{ij} = A_i L_i^{\alpha} \left[(\mu_j m_i)^{\gamma} (\tau_j H)^{1-\gamma} \right]^{\beta}, \quad \gamma < 1.$$

Centralized Solution

Define the total number of managers in the establishment as $M_1 = m_1 + m_h$. With a singleestablishment firm, and given that the HQ's location is fixed at location o, the demand for both types of workers and the firm's total profits is given by:

,

$$M_{1}^{I} = \left[\phi_{m} \cdot \mu^{\beta} w_{o}^{-\alpha} v_{o}^{\alpha-1} p A_{i}\right]^{\frac{1}{\kappa}} \left[1 + \frac{(1-\gamma)\mu}{\gamma}\right]$$
$$L_{1}^{I} = \left[\phi_{l} \cdot w_{o}^{\beta-1} \left(\frac{v_{o}}{\mu}\right)^{-\beta} p A_{i}\right]^{\frac{1}{\kappa}},$$
$$\Pi_{o}^{I} = \kappa \left[\phi_{p} \cdot w_{o}^{-\alpha} \left(\frac{v_{o}}{\mu}\right)^{-\beta} p A_{i}\right]^{\frac{1}{\kappa}} - r_{o},$$

where the superscript I indicates that the equations correspond to the solution of a single establishment firm, and $\kappa = 1 - \alpha - \beta$, and ϕ_m, ϕ_l, ϕ_p are constants. An interior solution exists as long as $\alpha + \beta \in (0, 1)$. Similar to the case with the fixed proportion managerial bundle, this solution shows us that single-establishment firms hire more of both types of workers when there are higher agglomeration economies, lower input prices or higher price for the final good.

Fragmented Solution

Define the total number of managers in each establishment as $M_1 = m_1 + m_h$ and $M_2 = m_2$. When the firm has two establishments, with HQ in o and satellite establishment in $j \in \{o, s\}$, the optimal demand for both types of workers at each establishment and the firm's total profits can be written as:

$$\begin{split} M_{1o}^{II} &= \left[\phi_m \cdot \left(\frac{\mu \mathbb{R}}{v_o}\right)^{(1-\gamma)\beta} pA_i\right]^{\frac{1}{\kappa}} \left[\left(\frac{\mu^{\gamma\beta}}{w_o^{\alpha} v_o^{1-\alpha}}\right)^{\frac{1}{1-\alpha-\gamma\beta}} + \frac{(1-\gamma)\mu \mathbb{R}}{\gamma v_o}\right], \\ M_{2j}^{II} &= \left[\phi_m \cdot \left(\frac{\mu \mathbb{R}}{v_o}\right)^{(1-\gamma)\beta} pA_i\right]^{\frac{1}{\kappa}} \left(\frac{\tau^{(1-\gamma)\beta}}{w_j^{\alpha} v_j^{1-\alpha}}\right)^{\frac{1}{1-\alpha-\gamma\beta}}, \\ L_{ij}^{II} &= \left[\phi_l \cdot \left(\frac{\mu \mathbb{R}}{v_o}\right)^{(1-\gamma)\beta} pA_i\right]^{\frac{1}{\kappa}} \left(\frac{\mu_j^{\gamma\beta} \tau_j^{(1-\gamma)\beta}}{w_j^{1-\gamma\beta} v_j^{\gamma\beta}}\right)^{\frac{1}{1-\alpha-\gamma\beta}}, \quad i \in \{1,2\}, \\ \Pi_{(o,j)}^{II} &= \kappa \left[\phi_p \cdot \left(\frac{v_o}{\mu}\right)^{-(1-\gamma)\beta} \mathbb{R}^{1-\alpha-\gamma\beta} pA_i\right]^{\frac{1}{\kappa}} - r_o - r_j, \end{split}$$

where ϕ_m, ϕ_l, ϕ_p are constants, the superscript II indicates that the equations correspond to the solution of a two-establishment firm, $\Pi_{o,j}^{II}$ denotes the profits of a firm with two establishments located in o and $j \in \{o, s\}$, respectively, and $\kappa = 1 - \alpha - \beta$. Moreover,

$$\mathbb{R} = \sum_{j \in \mathbb{L}} \left[\frac{\left(\tau_j^{1-\gamma} \mu_j^{\gamma} \right)^{\beta}}{w_j^{\alpha} v_j^{\gamma\beta}} \right]^{\frac{1}{1-\alpha-\gamma\beta}},$$

where \mathbb{L} is the set containing the locations of each of the firm's establishments. In the case of our two-establishment two-location model, \mathbb{L} equals either $\{o, o\}$ or $\{o, s\}$. The expression \mathbb{R} can be thought as a measure of the marginal revenue of an additional manager that works in the production of H. Comparing these equations with the respective equations from the text, it is evident that the comparative statics with respect to wages, communication costs and the agglomeration parameter hold.

C.4.1 Model Predictions

Labor composition within the establishments

The ratio of managers-to-workers for single-establishment firm is still given by:

$$\left(\frac{M_1}{L_1}\right)_{SE} = \frac{\gamma\beta}{\alpha} \frac{w_o}{v_o}.$$

Consider now the ratio of managers to workers at non-HQ establishments from multiestablishment firms. In this case, the ratio is the same as for single-establishment firms, and thus, does not depend on the managerial wage gap across locations:

$$\left(\frac{M_2}{L_2}\right)_{ME} = \frac{\gamma\beta}{\alpha} \frac{w_s}{v_s}.$$

The ratio of managers-to-workers at the HQ is given by:

/--·

$$\left(\frac{M_1}{L_1}\right)_{ME} = \frac{\gamma\beta}{\alpha} \frac{w_o}{v_o} \left\{ 1 + \left(\frac{1-\gamma}{\gamma}\right) \left[1 + \left(\tau^{(1-\gamma)\beta}\mu^{-\gamma\beta}\left(\frac{w_o}{w_s}\right)^{\alpha}\left(\frac{v_o}{v_s}\right)^{\gamma\beta}\right)^{\frac{1}{1-\alpha-\gamma\beta}} \right] \right\}.$$

Just as in the case with the fixed-proportions managerial bundle, this equation suggests that higher wages at the HQ, relative to the non-HQ establishment, would lead to an increase the manager-to-worker ratio at the HQ. Moreover, lower communication costs magnify these effects. We can also derive elasticities of the HQ ratio of managers-to-workers, with respect to changes in the wage gap of workers and managers across locations. These elasticities are:

$$\begin{aligned} \frac{\partial \log(M_1/L_1)}{\partial \log(w_o/w_s)} &= \frac{\beta(1-\gamma)}{1-\alpha-\gamma\beta} \cdot \frac{w_s L_2}{v_o M_1} > 0, \\ \frac{\partial \log(M_1/L_1)}{\partial \log(v_o/v_s)} &= \frac{\beta^2 \gamma(1-\gamma)}{\alpha(1-\alpha-\gamma\beta)} \cdot \frac{w_s L_2}{v_o M_1} > 0 \end{aligned}$$

Labor composition across establishments

Under our current assumptions, we can write the ratio of workers at the HQ, relative to the

non-HQ establishment, as:

$$\frac{L_1}{L_2} = \left(\tau^{-(1-\gamma)\beta}\mu^{\gamma\beta}\left(\frac{w_o}{w_s}\right)^{\gamma\beta-1}\left(\frac{v_o}{v_s}\right)^{-\gamma\beta}\right)^{\frac{1}{1-\alpha-\gamma\beta}}$$

Similarly, for managers:

$$\frac{M_1}{M_2} = \frac{w_o/w_s}{v_o/v_s} \left\{ 1 + \left(\frac{1-\gamma}{\gamma}\right) \left[1 + \left(\tau^{(1-\gamma)\beta}\mu^{-\gamma\beta}\left(\frac{w_o}{w_s}\right)^{\alpha}\left(\frac{v_o}{v_s}\right)^{\gamma\beta}\right)^{\frac{1}{1-\alpha-\gamma\beta}} \right] \right\}.$$

Note that the share of workers and managers at the HQ still depends negatively on the respective wage gap across locations. This result suggests that the size of the HQ relative to the non-HQ establishment decreases when the wage gap across locations is wider. From the two equations above, we can calculate the respective across-location elasticities of substitution, of workers and managers, as:

$$\begin{array}{lll} \displaystyle \frac{\partial \log(L_1/L_2)}{\partial \log(w_o/w_s)} & = & \displaystyle \frac{\gamma\beta-1}{1-\alpha-\gamma\beta} < 0, \\ \displaystyle \frac{\partial \log(M_1/M_2)}{\partial \log(v_o/v_s)} & = & \displaystyle -1 - \displaystyle \frac{\beta}{1-\alpha-\gamma\beta} \cdot \displaystyle \frac{v_o m_1}{v_o M_1} < 0, \end{array}$$

where $M_1 = m_1 + m_h$. While the elasticity of substitution of workers across establishments is constant, this is not the case for managers. The latter elasticity depends on the cost share of production managers on the total managerial costs at the HQ. In particular, if production is not dependent on HQ services, the elasticity converges to a constant given by $-1 - (\beta/1 - \alpha - \gamma\beta)$. Moreover, as the use on HQ services increases, this elasticity decreases and converges to -1. When comparing these two elasticities with the ones derived in the text for the fixed-proportions managerial bundle, an two important difference comes up: none of the above elasticities depend on the communication costs parameter τ . This theoretical result contradicts our result from Section 6.1, which suggest a significant interaction between changes in wages across locations and the distance between establishments. This evidence also suggests that the model with a fixed-proportion managerial bundle can fit the data better than the one using the Cobb-Douglas function.

C.5 Changes in HQ Productivity over Time

To include changes over time of the parameter representing the HQ productivity advantage, we derived our estimating equations (23), (21) and (22) to allow for changes in μ over time. Following the same assumptions and derivation procedure, the respective resulting equations can be written as:

$$d\log\left(\frac{M}{L}\right)_{ijhst} = -d\log\left(\frac{v}{w}\right)_{it} + \mathbf{1}_{\{HQ\}}\frac{\alpha(1-\omega_h)}{1-\alpha-\beta}d\log\left(\frac{w_h}{w_i}\right)_t + \left\{\mathbf{1}_{\{HQ\}}\frac{(1-\alpha)(1-\omega_h)(1-\gamma_i)}{1-\alpha-\beta} - \mathbf{1}_{\{N\}}\gamma_i\right\} \left[d\log\left(\frac{v_h}{v_i}\right)_t - d\log(\mu_t)\right] \quad (45)$$
$$- \left\{\mathbf{1}_{\{HQ\}}\frac{\beta-(1-\alpha)(1-\gamma_i)}{1-\alpha-\beta}(1-\omega_h) + \mathbf{1}_{\{N\}}\gamma_i\right\}\sum_t \mathbf{1}_t \cdot [c_t\log dist_{ih}] + d\delta_t + \zeta_{js} + d\epsilon_{ijhst},$$

$$d\log\left(\frac{L_h}{L_i}\right)_{st} = \frac{\beta - 1}{1 - \alpha - \beta} d\log\left(\frac{w_h}{w_i}\right)_t - \frac{\beta(1 - \gamma_i)}{1 - \alpha - \beta} \left[d\log\left(\frac{v_h}{v_i}\right)_t - d\log(\mu_t)\right] \\ + \frac{\beta\gamma_i}{1 - \alpha - \beta} \sum_t \mathbf{1}_t \cdot [c_t \log dist_{ih}] + d\delta_t + \zeta_{js} + d\eta_{ijhst},$$
(46)

$$d\log\left(\frac{M_h}{M_i}\right)_{st} = \frac{(\alpha-1)(1-\gamma_i)\omega_h}{1-\alpha-\beta} \left[d\log\left(\frac{v_h}{v_i}\right)_t - d\log(\mu_t)\right] - \frac{\alpha\omega_h}{1-\alpha-\beta}d\log\left(\frac{w_h}{w_i}\right)_t - \left[\frac{(\alpha-1)\gamma_i\omega_h}{1-\alpha-\beta} - (1-\omega_h)\right]\sum_t \mathbf{1}_t \cdot [c_t\log dist_{ih}] + d\delta_t + \zeta_{js} + d\nu_{ijhst}, \quad (47)$$

where all the notation follows the notation in the text, except for the term $d \log(\mu_t)$, which corresponds to the changes over time of the logarithm of HQ-productivity advantages. Since we cannot observe these parameters, we estimate them as a series of 5-year period fixed effects.

D Extensive Margin: Number of Establishments

We are also interested in knowing how changes in wages, communication costs and agglomeration economies affect other dimensions of the observed firm fragmentation patterns. Since our model gives us sharp predictions regarding the firm's number of establishments, we focus on this appendix on the extensive margin decisions. In particular, we use numerical simulations to analyze whether our model captures some of the important regularities we find in the data.

We start by analyzing how distance from HQ, wage differentials and population density affect the probability that a firm has an establishment in a given municipality. Table A8 presents probit regressions between a binary variable that equals 1 if a firm with HQ in k has an establishment in municipality j and different combinations of our variables of interest. Wages are measured following the methodology described in Section 2, but include a municipality-year fixed effect in the regressions instead of an occupation-establishment-year fixed effect. Thus, the measured wage is the average of all firms in a municipality and is the same for both workers and managers. All of the regressions include sector, year and HQ commuting area fixed effects. We have not included firm fixed effects to avoid problems with incidental parameters. Identification of these estimates comes from comparing, in the cross-section, the location choices of firms within the same sector, HQ commuting area, or year. The estimates presented in this table are merely correlations and do not represent causal relations.

	Prob(Firm with HQ in k has an est in j)					
	(1)	(2)	(3)	(4)	(5)	(6)
Variables \setminus Period	1994-2016	1994-2016	1994-2016	1994-2001	2002-2008	2009-2016
Dist to HQ	-0.44***	-0.46***	-0.50***	-0.50***	-0.50***	-0.49***
	(0.026)	(0.023)	(0.023)	(0.025)	(0.024)	(0.021)
$log(W_k/W_j)$		1.20^{**}	-12.12***	-11.55***	-12.68^{***}	-12.44^{***}
		(0.610)	(2.575)	(2.505)	(2.618)	(3.158)
$log(W_k/W_j)$ *Dist to HQ			2.77^{***}	2.71^{***}	2.80^{***}	2.86^{***}
			(0.529)	(0.518)	(0.531)	(0.636)
Pop Density at HQ			0.00	0.01	0.01	-0.01
			(0.028)	(0.028)	(0.031)	(0.028)
Ν		3,895,617		1,088,243	1,262,067	1,388,264

Table A8: Location Probability

Notes: Robust standard errors in parentheses. All regressions include firm sector, year and HQ commuting area fixed effects. Regressions were estimated using a probit model. Numbers correspond to estimated parameters, not to marginal effects. Wages are measured following the methodology described in Section 2, but include a municipality-year fixed effect instead of an occupation-establishment- year fixed effect. *** p < 0.01, ** p < 0.05, * p < 0.1.

Column (1) shows that firms are more likely to have an establishment in municipalities that are closer to the firm's HQ. Column (2) shows that this probability also increases if the wages in the municipality are lower than the wages at the HQ's location. The interaction between relative wages and distance from Column (3) indicates that a lower relative wage offsets the impact of distance: A firm might be willing to have an establishment in a location far from its HQ if it offers a big enough cost advantage. However, if a municipality is farther away and is also more expensive, the probability of locating there is even lower. We do not find any significant correlation between firm location and population density. In Columns (4) to (6) we split the sample into three time periods and the results are almost identical. In Table A9, we estimate similar regressions but use a negative binomial model and the number of establishments in each municipality as the dependent variable. The results of these regressions point to the same intuition.

	Number of Establishments in k of a Firm with HQ in l					
	(1)	(2)	(3)	(4)	(5)	(6)
Variables \setminus Period	1994-2016	1994-2016	1994-2016	1994-2001	2002-2008	2009-2016
Dist to HQ	-0.84***	-0.85***	-0.86***	-0.89***	-0.86***	-0.84***
	(0.039)	(0.037)	(0.036)	(0.036)	(0.036)	(0.035)
$log(W_l/W_k)$		2.16^{*}	-19.28^{***}	-18.51^{***}	-20.19^{***}	-19.77^{***}
		(1.310)	(4.371)	(4.218)	(4.445)	(5.285)
$log(W_l/W_k)$ *Dist to HQ			4.54***	4.53***	4.57***	4.62***
			(0.980)	(0.955)	(0.966)	(1.149)
Pop Density at HQ			-0.01	-0.01	0.01	-0.01
			(0.047)	(0.042)	(0.050)	(0.051)
N		3,895,617		1,088,243	1,262,067	1,388,264

Table A9: Number of Establishments

Notes: Robust standard errors in parentheses. All regressions include firm sector, year and HQ commuting area fixed effects. Regressions were estimated using a negative binomial model. Wages are measured following the methodology described in Section 2, but include a municipality-year fixed effect instead of an occupation-establishment-year fixed effect. *** p < 0.01, ** p < 0.05, * p < 0.1.

To analyze whether our model captures this intuition, we analyze numerically what happens with the number of establishments and their locations when wage differences across locations, communication costs and agglomeration economies change. We also consider the case in which the firm can open a third establishment, keeping fixed the number of locations (o,s). For these cases, we follow equation (31), which is discussed in Appendix C.1. Table A10 shows the value of the parameters used for the simulations and its source. These parameters are either estimated in Section 6.2 or are taken from the data.

1. Communication Costs and Wage Gaps

Consider first what happens with the optimal number of establishments and their locations when communication costs and wage gaps across locations change. Define $g_l \equiv w_o/w_s$ and $g_m \equiv v_o/v_s$, with g_l or g_m larger than one if the wages of workers or managers are higher at the firm's HQ relative to location s. In Panel A of Figure A6, we fix the wage gap of managers g_m and allow τ to vary between 0 and 1 and the wage gap of workers g_l to vary between 0.8 and 1.3. In Panel B, we fix g_l and allow g_m to vary between 0.8 and 1.3. In Panel C, we fix τ and allow g_m and g_l to vary between 0.8 and 1.3. In all simulations, Roman numerals in the legend denote the firm's total number of establishments, with their locations in the subsequent parentheses.

Panels A and B show illustrate important insights. First, notice that for a given level of wage gap (regardless of the type of worker), lower communication costs $(\uparrow \tau)$ lead to a higher number of establishments-that is, higher fragmentation. Second, firms are more likely to be fragmented, and are more sensitive to reductions in communication costs, when wages in location o are higher than wages in s. Finally, Panel C shows that when workers and managers are relatively cheap at the HQ location, the firm will choose to be in a centralized equilibrium. However, as location s becomes cheaper, the firm will start opening more establishments outside the central location. This effect is stronger if wage differences increase for both types of workers. Finally, note that under our current set of parameters, if the wage of production workers is very low in s relative to o, the firm will open establishments in s, almost regardless of the managerial wage gap.

Parameter	Value	Description	Source					
I. Productio	I. Production Function							
α	0.447	Worker share	Table 5					
β	0.340	Managerial bundle share	Table 5					
λ	0.579	Leontief coef. of local mgr.	Average wage costs of production managers rela-					
			tive to total managerial wage costs.					
heta	1.0	Leontief coef. of τH	Normalization					
II. Prices								
p	3.6	Price of final good						
v_o	1.075	Managerial wage at o	Exponent of the coefficient of the 1994 HQ					
-		0 0	dummy from a wage equation (similar to Equa-					
			tion (1) for managers.					
w_o	0.986	Worker wage at o	Exponent of the coefficient of the 1994 HQ					
			dummy from a wage equation (similar to Equa-					
			tion (1)) for workers.					
g_m	1.043	Managerial wage gradient	Ratio of v_o and v_s , which is built equivalently but					
			using the 1994 non-HQ dummy					
g_l	0.985	Worker wage gradient	Ratio of w_o and w_s , which is built equivalently					
			but using the 1994 non-HQ dummy					
f_s	1.0	Fixed cost at s	Normalization					
ϕ	0.813	Fixed cost gradient	90th-10th ratio of municipality level hedonic price					
			indices for 1994.					
III. Other								
A_i	1.0	Firm Productivity	Normalization					
μ_{o}	1.065	Agglomeration econ. in o	Copenhagen wage premium, computed from a					
, 0			wage equation similar to Equation (1), but with					
			worker fixed effects.					
μ_s	1.0	Agglomeration econ. in \boldsymbol{s}	Normalization					

Table A10: Parameters for Simulations

This table shows the parameters used in Figures A6 to A10, their descriptions and source. More detailed information is available on request

2. Communication Costs and Agglomeration Economies

Consider now what happens with the optimal number of establishments when communication costs and managerial agglomeration economies change. In particular, we plot the number of establishments chosen by the firm for different values of $\tau \in [0, 1]$ and $\mu \in [0.75, 2]$. Panel A in Figure A7 shows the scenario in which the firm chooses between being a centralized firm (with HQ in o) or opening a second establishment. In this particular case, for lower levels of agglomeration economies in location j = o, cheaper communication costs would lead the firm to move from a centralized to a fragmented equilibrium. For higher levels of agglomeration economies, cheaper communication costs can cause the movement of the non-HQ establishment from the central to the satellite location. In either case, a decrease in communication costs leads to spatial decentralization.

On the other hand, and perhaps quite counterintuitive, is the relation between the number of establishments and the agglomeration parameter μ . Note that for some levels of communication costs, higher μ leads to the creation of a second establishment in s. This is because when managerial agglomeration economies increase, the firm would like to hire more managers and

Figure A6: Communication Costs and Wage Gaps



These figures show simulations of the firm's optimal number of establishments and their locations, when wages across locations and communication cost change. The simulations are based on equations (24) and (29). Roman numerals in the legend denote the firm's total number of establishments, with their locations in the subsequent parentheses.

workers to produce more final good and HQ services. However, given the existence of diminishing marginal returns, the marginal benefit of this increase in the total number of employees might not be larger than the marginal cost, unless the firm opens a second establishment. Thus, the firm might opt for the fragmented equilibrium if communication costs are low enough. As agglomeration becomes even stronger, the firm would like to move the second establishment back to the central location.

In Panel B we analyze the case in which the firm can open a third establishment in either of the two locations. In this case, for a given value of communication costs, higher agglomeration economies lead the firm to move from having one, to having two, to having three establishments. A similar intuition applies if communication costs decrease, for a wide range of levels of agglomeration economies. Finally, if agglomeration economies are particularly strong, the firm always chooses to have three establishments regardless of the level of communication costs, with most (if not all) in the central location.

In Figures A8 and A9, we show that the average number of establishments per firm and the average distance to the HQ increases as communication costs decrease. These figures resemble the empirical results from Acosta and Lyngemark (2021).

Finally, we explore what happens to the optimal number of establishments when firm productivity and communication cost change. Panels A and B from Figure A10 show that at high levels of communication costs, only highly productive firms open additional establishments, and these establishments are in the same location as their HQ. Furthermore, as communication costs decrease, a group of firms in the middle of the productivity distribution also decide to open additional establishments, mostly in s, where both land and labor are cheaper.



Figure A7: Communication Costs and Agglomeration Economies

These figures show simulations of the firm's optimal number of establishments and their locations, when communication cost and agglomeration economies change. The simulations are based on equations (24) and (29). Roman numerals in the legend denote the firm's total number of establishments, with their locations in the subsequent parentheses.



Figure A8: Average Number of Establishments and Communication Costs



Figure A9: Distance to HQ and Communication Costs

Figure A10: Firm Productivity and Communication Costs



These figures show simulations of the firm's optimal number of establishments and their locations, when communication cost and firm productivity change. The simulations are based on equations (24) and (29). Roman numerals in the legend denote the firm's total number of establishments with their locations in the subsequent parentheses.