Spatial Wage Differentials, Geographic Frictions and the Organization of Labor within Firms

Camilo Acosta* Ditte Håkonsson Lyngemark†

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Abstract

This paper studies the spatial organization of firms, both theoretically and empirically. Two new facts in Danish register data motivate the analysis. First, firms have become more fragmented over time. Second, headquarters (HQ) establishments have become more-manager intensive, despite a significant increase in managerial wages at HQ locations. We study the roles of spatial wage disparities and communication costs in explaining these two trends. Immigration shocks are the source of identifying variation for changes in relative labor supply. We estimate elasticities of substitution across establishments within firms of 3.5 for workers and 0 for managers. Estimates indicate that increases in the wage of managers at the HQ relative to non-HQ locations, explain 66% of the increase in HQ managerial intensity. This can be explained by the associated increasing demand for headquarter services and managerial knowledge as satellite establishments become larger.

JEL: D22, J23, L22, L23, R30.

Keywords: firm organization, multi-establishment firms, wages, communication costs, agglomeration.

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†Office of Housing and Construction, the Danish Social and Housing Authority, Denmark.

1 Introduction

Multi-establishment (ME) firms and large business groups play a crucial role in developed economies. For instance, half of all private-sector employment in Denmark is generated by ME firms (7% of the total), while in France they hold 60% of total private employment (Charnoz et al., 2018). Moreover, these shares have been increasing significantly over the last decades in different high-income countries, such as the United States (Kim, 1999), France (Charnoz et al., 2018), and Denmark (Acosta and Lyngemark, 2021). In addition, these firms typically locate their headquarters (HQ) in denser urban areas (Strauss-Kahn and Vives, 2009) and, through their network of establishments, they propagate local economic shocks across space (Giroud and Mueller, 2019; Gumpert et al., 2022). Therefore, the organization of these large firms could be behind the observed sorting of high-skill workers into high-wage cities, and other changes in local real estate and labor markets (Acosta and Lyngemark, 2021; Spanos, 2019).

This paper studies how regional wage disparities and geographic frictions, such as communication costs, and agglomeration economies, affect the spatial organization of firms. Particularly, we consider how these forces drive the creation of new establishments, establishment location choices, and the allocation of labor across them, within multi-establishment firms. Our analysis proceeds both theoretically and empirically, concluding with one of the first structural analysis of these issues in the literature. Among other results, we find that firms are more likely to substitute workers, relative to managers, out of their HQ into satellite establishments. Moreover, most of the observed changes in the organization of labor within firms can be explained by the existence of intangible inputs, such as HQ services and managerial knowledge, together with the increasing costs of managers at HQ locations.

Our starting point is the dramatic changes in the internal spatial organization of firms along two dimensions. First, firms have become more fragmented over time. In Acosta and Lyngemark (2021), we show that the number of establishments per firm in Denmark increased by 21% between 1981 and 2016, while the average distance between establishments and their headquarters (HQ) doubled.[1] Second, during the same period, HQs became more manager intensive relative to other establishments, despite a significant relative increase in managerial wages in HQ locations. Specifically, between 1996 and 2011, the ratio of managers to workers increased by 16% at firms' HQ, relative to satellite establishments, while the wage of managers at the HQ, relative to non-HQ, increased by 19%.

To understand these facts, we first build a model of firm location and labor demand. In the model, a firm chooses the number of establishments and the location and labor composition of each. This model builds on the literature on multinational firms (Helpman, 1984; Horstmann and Markusen, 1987; Markusen, 2002). The firm's HQ establishment, in addition to producing a final good, produces a within-firm public good, which we call HQ services. This good is manager intensive and can be used in all establishments, but with some geographic attenuation due to communication costs. Since this good is non-rival, it generates within-firm economies

[1] The increasing degree of spatial decentralization within firms have also been documented for France (Charnoz et al., 2018) and the United States (Davis and Henderson, 2008).

of scale that encourage the creation of multiple establishments. Recent literature has shown the importance of internal transfers of such intangible inputs in explaining the integration of multiple establishments and the productivity of large firms (Atalay et al., 2014). Moreover, wage differences between a firm's HQ and other locations create an incentive for the firm to open new establishments and substitute workers out of the HQ to places where wages are relatively low. By opening new establishments, firms incur extra costs of production and communication. Both of these forces limit the number of establishments a firm might have.

We use this model to study the roles of changes in wages across locations and communication costs in determining the allocation of factors across establishments. Our model suggests that increasing wage differences between HQ and non-HQ establishments lead to an increase in the size of the latter. This increase in size generates an increase in the demand for HQ services, and thus in the relative demand for managers at HQ. Therefore, an increase in the price of managers at HQ locations could lead to a more manager-intensive HQ due to the existence of these HQ services. In line with this result, our model also predicts that the elasticity of substitution across locations is larger for workers than for managers. From the model, we derive a system of relative labor demand equations that allows us to estimate the model's structural parameters.

Our empirical work uses matched employer-employee register data for Denmark between 1986 and 2016. Using these data, we can determine the firm's number of establishments and their locations. Furthermore, these data allow us to characterize every establishment in terms of the occupations of its workers; in particular, managers and workers. For econometric identification, we use an augmented version of the standard immigration share shifter instrument widely used in the literature (Foged and Peri, 2016; Lewis, 2011; Baum-Snow et al., 2018). Since we seek labor supply shocks at establishments' locations, we map these immigration shocks from municipalities of residences into workplace municipalities using historical commuting flows. We use this instrument as a source of exogenous variation across and within municipalities for changes in the supply of managers and workers. To the best of our knowledge, the use of this commuting-augmented immigration instrument constitutes a novel identification strategy in this literature.

As an initial validation of the model and as a way of understanding the labor substitution patterns within firms, we estimate reduced-form relative labor demand equations. We find a within-establishment elasticity of substitution between managers and workers of around 2.4. Moreover, we find that this elasticity is larger for single-establishment (SE) firms than for establishments belonging to ME firms. The latter group is less sensitive to within-establishment wage differences, as they can also substitute labor across locations. We also estimate across-location relative labor demand equations for both workers and managers, separately. Our estimates suggest that on average, firms respond more to differences in the wage of workers across locations (elasticity of -3.5), than to differences in the wage of managers (elasticity of 0). These difference suggest important managerial economies of scale within firms. We also find that these across-establishments elasticities of substitution are stronger for closer locations compared to farther ones, with this relation becoming stronger over time. We interpret this result as evidence supporting a complementary relation between local managers and lower communication costs.

With this validation, we proceed to estimate the model structurally. Doing so allows us to back out the relative importance of within- and across-establishment wage changes and changes in communication costs in explaining the observed increase in the manager-to-worker ratio at HQ relative to non-HQ establishments. Our results suggest that the standard labor demand channel, i.e., changes in the wages of managers relative to the wage of workers, account for -5% of the observed increase in HQ managerial intensity. In other words, if we assume that firms and establishments are equivalent, we would not be able to explain changes in managerial intensity successfully. On the other hand, around 66% of the total change can be explained by rising wages of managers in HQ relative to non-HQ locations. These results emphasize the importance of firm-level scale economies interacted with rising relative wages for managers in HQ locations, as suggested by the model. Finally, changes in communication costs explain around 40%.

Our paper is related to five broad strands of literature. First, by studying the organization of workers and managers within firms, this paper contributes to the literature studying the organization of firms (Radner, 1992; Becker and Murphy, 1992; Garicano, 2000; Bresnahan et al., 2002; Bloom et al., 2014). This literature studies the relation between communication costs, firm organization and knowledge hierarchies, and the importance of managers for processing information and solving problems. Our paper also fits into a recent empirical literature that studies internal input markets in ME and vertically integrated firms (Giroud, 2013; Atalay et al., 2014; Behrens and Sharunova, 2015; Tate and Yang, 2015; Charnoz et al., 2018; Cestone et al., 2022; Gumpert et al., 2022). This literature mostly studies the effect of reductions in communication costs on establishment-level outcomes, focusing on the proper econometric identification of different predictions drawn from the theory. Closely related to our paper, Charnoz et al. (2018) and Gumpert et al. (2022) study the impact of reductions in geographic frictions on firm organization in France and Germany, respectively, using the introduction of high-speed train routes. Similarly, Jiang (2022) finds how better internet access in the US lead to the creation of new establishments by manufacturing firms.

Our paper contributes to this research in three ways. First, we propose changes in spatial wage disparities as a new mechanism that affects the internal spatial organization of firms. Moreover, we show that this mechanism can be critical for determining changes in internal labor markets, establishment specialization and location decisions. Second, we show that there exist important interactions between these regional wage disparities and communication costs. Third, to the best of our knowledge, this is the first paper to specify and estimate a structural model of the spatial organization of labor within firms, that includes an intangible input such as HQ services.

Second, within urban economics, this paper builds on the papers that studies the location decisions of multi-unit firms, including Ota and Fujita (1993); Duranton and Puga (2005); Rossi-Hansberg et al. (2009); and Gokan et al. (2019) on the theoretical front, and Henderson and Ono (2008); Davis and Henderson (2008); Bartelme and Ziv (2017); and Oberfield et al. (2020) on the empirical side. Both strands of this literature have developed independently, partially due to the lack of the appropriate data needed to estimate these models. On one hand, theoretical articles have focused on showing how reductions in communication costs lead to firm fragmentation and to changes in the distribution of activities within and across cities. On the other hand, empirical studies have focused mainly on the determinants of headquarters location and agglomeration. This paper unifies these literatures by building a firm-level model that can be estimated using detailed establishment-level data, allowing evaluation of the relative importance of changes in spatial wage differentials and communication costs in explaining observed patterns.

Third, our paper also contributes to the literature on spatial sorting. In particular, this literature has found an increasing sorting of high-skilled workers and high-productivity firms into high-wage geographic areas (Moretti, 2004; Combes et al., 2008, 2012; Diamond, 2016; De La Roca and Puga, 2017; Baum-Snow et al., 2018; Gaubert, 2018; Eckert, 2019). In this paper, we show that similar sorting patterns are occurring within firms, with headquarters establishments becoming more manager intensive despite significant increases in the wage of managers in HQ relative to satellite establishments. As HQ are disproportionately located in denser labor markets with more rapidly rising managerial wages, this increase in within-firm polarization could be contributing to the spatial polarization across labor markets (Acosta and Lyngemark, 2021).

Four, there have also been advances in the international trade literature on multinational enterprises. Most of this literature focuses on the choice between exporting, outsourcing and offshoring, and has considered differences in market access and production and transportation costs to be the main determinants (Helpman, 1984; Markusen, 2002; Antràs et al., 2006; Antràs and Yeaple, 2014; Fort, 2017; Tintelnot, 2017). We consider the rise in multinationalization to be an extreme case of the firm fragmentation process we study. In particular, our work relates to the theoretical work of Antras et al. (2006), who provide a model of team formation in an offshoring setting.

Finally, this paper relates to an extensive literature in management science studying the organization of modern corporations and the role of headquarters. Articles in this literature usually focus on one of two cases: multibusiness or multinational firms (Menz et al., 2015). On one hand, the multibusiness approach studies firms that are active across different product market within national borders (Chandler, 1969; Fligstein, 1985; Chandler, 1991; Collis et al., 2007). On the other hand, articles studying multinational firms (from this management perspective) deal with firms operating across several geographic markets (Laamanen et al., 2012; Collis et al., 2012). This paper contributes to this literature by introducing a framework that applies some of the elements prevalent in the multinational firm approach (i.e., firm location and geographic differences) into the labor demand decision of multiunit firms.

The rest of the paper proceeds as follows. In Section 2, we present our data and describe our sample. In Section 3, we show the increase in managerial intensity at firms' headquarters, relative to non-HQ establishments. In Section 4, we develop our theoretical model and discuss the main comparative statics. Section 5 presents the empirical and identification strategies, while Section 6 presents our empirical results and numerical exercises. Section 7 concludes.

2 Data Description

Our data contain the full population of employers and employees collected by Statistics Denmark (DST) between 1981 and 2016. We give a more detailed description of these data in Acosta and Lyngemark (2021), but describe here the main elements. We use establishment records from the Integrated Database for Labor Market Research (IDA), which contains all matches between employees and workplaces every year. We match these data with the General Firm Statistics and the Firm Accounting Statistics. Since the data have unique firm and establishment identifiers, we can determine whether the firm has one or multiple establishments.

We observe the municipality and the traffic zone of each establishment, as well as the municipality reported by the firm in its accounting records.[2] Hence, we compute the distance between two establishments as the distance between their respective traffic zones. DST does not provide information about which of the firm's establishments is the headquarters (HQ). Therefore, we categorize an establishment as the HQ if its municipality is the same as the one reported by the firm and has at least five employees.[3]

The IDA also contains the hourly wage, tenure and experience of each worker. We deflate wages using the 2015 national Consumer Price Index. We keep the employee's main job which is defined by DST based on the worker's main source of income. Moreover, we link our data with registers containing data on the worker's age, immigration and education, from the Population, Immigration and Education Statistics registers, respectively. We keep workers between 15 and 80 years old. Since we can match employers and employees, we can also compute commuting flows between municipalities by year and occupation. As will become clear in Section 5, we use these commuting flows together with immigration shocks for our identification strategy.

To distinguish between managers and non-managers, we use two variables: (i) PSTILL/P-SOC from the IDA register, which defines the primary job for each worker in terms of their position, and (ii) DISCO88 from the Labor Classification Module (AKM).[4] We categorize an individual as a manager if she is categorized as a manager or top executive by either classification. Nevertheless, there is a significant share of establishments for which we cannot identify any manager. As will be evident in Section 4, our model requires that all establishments have a positive number of workers and managers. Therefore, for every establishment with no *visible* manager, we assign the worker(s) with the highest hourly wage and highest completed educational level as the manager.

[2]Traffic zones are geographic areas smaller than municipalities and are defined by the National Transport Model (LTM) developed by the Technical University of Denmark. There are 907 zones in Denmark. The LTM also contains the distance and travel time matrix for each pair of zones, including an approximation for the diagonal of these matrix.

[3]Using this definition, we define a HQ establishment for 96% of the firms in our sample. The remaining 4% are mostly firms that have more than one establishment in the same municipality as the one reported in their accounting records. In these cases, we take the establishments' labor composition into consideration and choose the establishment with the largest number of (i) managers, (ii) high wage earners, (iii) workers with master's or doctorate degree, and (iv) workers with technical or bachelor's degree.

[4]The DISCO classification changes in the registers between 2009 and 2010 from DISCO88 to DISCO08. These changes affected mostly the three- and four-digit-level codes, while we use mostly two-digit occupations. More information on the crosswalk used is available upon request.

Put together, all of these different registers imply that we can: (i) follow every establishment throughout its life, (ii) link it to a firm and determine whether it belongs to a multi-establishment firm, (iii) observe its location, and (iv) characterize its workforce in terms of occupations and other worker attributes. We limit our analysis to firms in the manufacturing, transportation, business services, and finance, insurance and real estate (FIRE) sectors, which account to 53% of Denmark's national employment between the last four decades. We follow the same sample criteria as in Acosta and Lyngemark (2021).[5] Since our identification strategy uses data between 1986 and 1994, we use data from 1994 onward for most estimations.

Moreover, to obtain consistent measures of establishment-occupation-year level wages, we must further restrict our sample. We keep only workers with hourly wages between 50 and 8,000 DKK.[6] and drop those whose reported wage has strong measurement error, as indicated by the data. Then, we drop microfirms (i.e., those with less than 10 workers) and those that were microfirms for at least 66% of our sample. We do not think this brings selection biases as 99.4% of these firms have only one establishment and this paper studies ME firms. Finally, we drop firms and establishments that only appear 1 year in our database (*singletons*), as these would only shrink our standard errors.

Table A1 presents descriptive statistics for this sample of firms. In Section C from the Online Appendix, we briefly discuss the spatial distribution of workers and HQ in Denmark. In particular, we show that the commuting areas of the four main municipalities account for 69% of HQ of ME establishments, and 51% of workers.[7] This concentration of HQ establishments is not unique to Denmark. Strauss-Kahn and Vives (2009) show that HQ establishments in the United States are disproportionally located in denser and the largest metropolitan areas.

We categorize every worker as a manager or non-manager. We do this aggregation regardless of individual characteristics and, thus, there is a high degree of heterogeneity within each category. This is particularly true for non-managers, since it encompasses different occupations such as secretaries, laborers and engineers. In order to make workforces comparable across establishments, we estimate a quality-adjusted average wage for each establishment and year by regressing the logarithm of the raw hourly wage of a worker p of occupation o (manager or non-manager, separately) in establishment i located in j at time t ($\log \omega_{pojkt}$) on a vector of observable characteristics and establishment-year fixed effects, as follows:

$$\log \omega_{poijt} = X_{pot}\beta^{o} + \log w_{oijt} + \varepsilon_{poijt}, \tag{1}$$

where the vector X_{pot} includes both time-variant and time-invariant worker characteristics including gender, region of origin, education category interacted with a polynomial of degree two of experience and tenure, detailed occupation and job position dummies; $\log w_{oijt}$ denotes an

[5] We dropped (i) establishments with no reported location, (ii) those located in sparsely populated islands, (iii) firms that had fewer than 4 employees for more than 66% of their existence in our data, (iv) establishments with 1 or 2 employees or that only appear one year, (v) firms with more than 99 establishments or that exhibit large jumps in the total number of establishments across years (mostly outliers in the data from 1985 to 1988).

[6] This corresponds to hourly wages between 7 and 1,164 in 2015 U.S. Dollars.

[7] Commuting areas are defined by Nielsen (2005) based on commuting flows across municipalities from 2004. We present them in Figure A1 in the Appendix.

establishment-year fixed effect for workers of occupation o. We use the (exponential of the) predicted values of these fixed effects $\hat{w}_{oijt}$ as the measure of establishment-level quality-adjusted wages in all of our empirical exercises. Finally, ε_{poijt} represents the error term. Besides from the quality-adjusted wages, from estimating the previous equations we can also calculate the efficiency units of each occupation used in each establishment i at time t.[8]

Results of the estimation of equation (1) can be found in Table A2. All these sample restrictions mean that each year we have on average 8,918 firms and 12,553 establishments, which contain around 622,000 workers. In total for the entire period, we have approximately 187k firm-year, 264k establishment-year, and more than 13 million worker-year observations.

3 Changes in the Spatial Organization of Firms

In Denmark in 2016, approximately 7% of all firms in the private sector had more than one establishment. These firms generated approximately 47% of all employment in the private sector and 54% of the total output. Moreover, the share of multi-establishment (ME) firms was 3.3% in 1981. Most of the increase in the share of ME firms has been driven by an increase in the number of ME firms, rather than by a decrease in the number of single-establishment firms. Most of these changes have come from changes in the internal spatial organization of firms.

In Acosta and Lyngemark (2021), we document an increase in the spatial decentralization of firms along several measures. First, we document an increase of 21% in the average number of establishments per firm and around 200% in the average distance of establishments to their HQ, for firms in the manufacturing and service sectors between 1981 and 2016. Moreover, we show that the average share of employment held at firms' HQ decreased by 13 percentage points. This last fact implies that a large number of jobs are being moved out of firms' HQ to other establishments within the firm. However, in that paper we are silent about the relations between these dynamics and spatial price differentials.

If a large number of jobs are being relocated–and since multi-establishment firms generate a disproportionate share of total employment–it is important to know how the distribution of different types of workers across establishments within firms has changed. For example, assume there is an exogenous shock to wages that positively affects headquarters' relative demand for managers. Since headquarters are disproportionally located in the main municipalities (as shown in Figure A5), firms' labor demand decisions could be contributing to the sorting of high-skilled people into large cities that has been found in the literature.

To explore how managerial intensity patterns have changed across establishments within firms, we compute the ratio of managers to workers (M/L) at each establishment i from firm f

[8] These efficiency units can be computed as $M^E_{ijt} = \sum_{p\in(i,m)} e^{X_{pmt}\hat{\beta^m}}$ for managers, and $L^E_{ijt} = \sum_{p\in(i,l)} e^{X_{plt}\hat{\beta^l}}$ for workers, where $\hat{\beta^o}$ denotes the estimated parameters from equation 1 and $\sum_{p\in(i,o)}$ denotes the sum across all workers within an establishment and occupation.

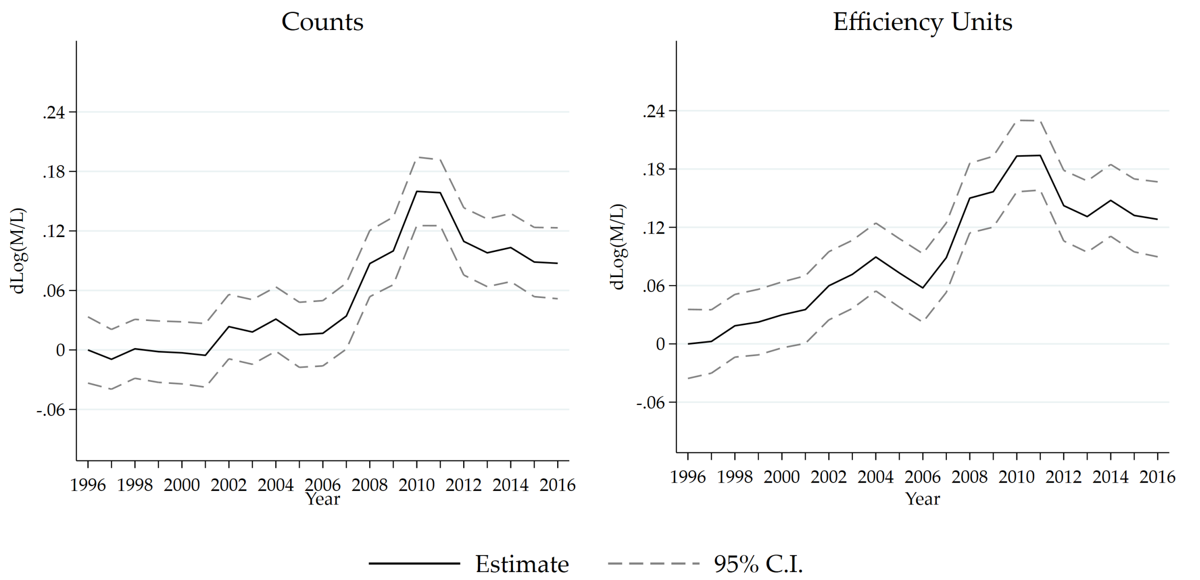
at time t, and estimate the following regression for multi-establishment firms:

$$\log\left(\frac{M}{L}\right)_{it} = \alpha_i + \delta_t + \lambda_{HQ_i,t} + \varepsilon_{it}, \tag{2}$$

where α_i are establishment fixed effects, δ_t time fixed effects, and $\lambda_{HQ_i,t}$ are HQ-specific time fixed effects. We also control for commuting area*sector trends in order to account for unobservable factors that could be driving these changes at the region and sector level. Figure 1 shows the estimated $\lambda_{HQ_i,t}$, which correspond to the difference in changes in the manager-to-worker ratio at HQ relative to non-HQ establishments. We plot this figure for the ratio both in terms of counts and in terms of efficiency units of labor, as computed in Section **??**.

Figure 1: Ratio of Managers to Workers

HQ relative to non-HQ, changes relative to 1996.



This figure shows the HQ*year fixed effects from a regression of the log ratio of the establishments' managers to workers on establishment, year and HQ*year fixed effects. The left panel computes the ratio in terms of raw counts. The right panel computes the ratio in terms of efficiency units.

In the left panel, we observe that between 1996 and 2011, the ratio of managers to workers increased by 16% at firms' HQ relative to non-HQ. Since the financial crisis, this ratio has slightly decreased. However, in 2016 the ratio was still 9% significantly larger than in 1996. The changes are larger when we examine them in terms of efficiency units of labor in the right panel. In this case, the ratio goes up by around 19% between 1996 and 2011, and then stabilizes around 13% after the crisis. One important concern is that the entry and exit of firms might be driving these results. For this reason, we estimate the same regression for a balanced panel of establishments. Figure A2 presents the results and shows that the change in this ratio is larger than the one computed for the whole sample, although the confidence interval is wider. For these firms, there was an increase of around 27% between 1996 and 2011 in the managerial intensity at HQ relative to non-HQ establishments. Another possibility is that the growth of multinational firms (MNEs) is driving our results. Therefore, we exclude those firms that reported an establishment outside Denmark. In Figure A3 we show that the upward trend is even stronger when we exclude MNEs.

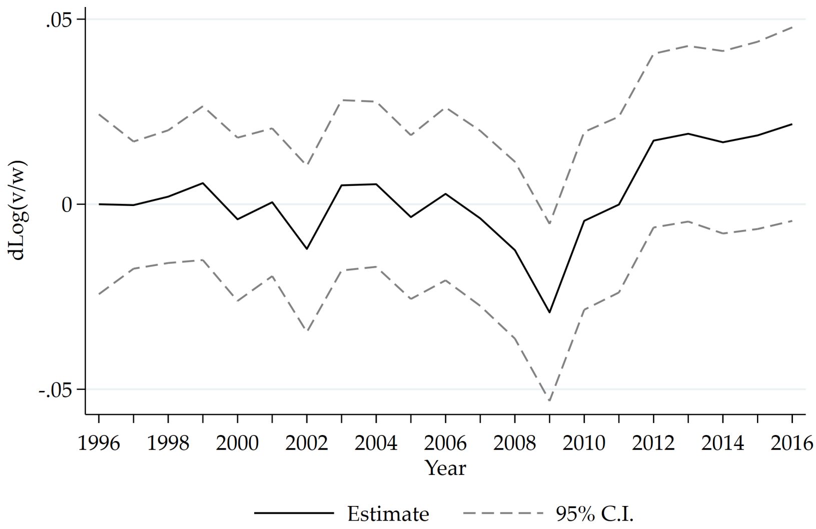
The increase in the ratio of managers to workers at firms' HQ starts to become puzzling when we look at the evolution of the respective wage ratio. In particular, equivalent to the regression from equation (2), we estimate a regression of the wage ratio of managers to workers at HQ, relative to non-HQ establishments, on the same set of fixed effects:

$$\log\left(\frac{v}{w}\right)_{it} = \alpha_i + \delta_t + \lambda_{HQ_i,t} + \varepsilon_{it}, \tag{3}$$

where v corresponds to the wage of managers and w to the wage of workers. These wages correspond to the estimated fixed effects from equation (1) for both occupations. Figure 2 shows the estimated $\lambda_{HQ_i,t}$. As can be seen, even though there has been a slight decrease in this ratio, the difference is not statistically different from zero for most of the period. Therefore, something other than standard labor demand mechanisms is likely to be behind the changes in managerial intensity.

Figure 2: Wage Ratio of Managers to Workers

HQ relative to non-HQ, changes relative to 1996.



This figure shows the HQ*year fixed effects from a regression of the log ratio of establishments' wages of managers to workers on establishment, year and HQ*year fixed effects.

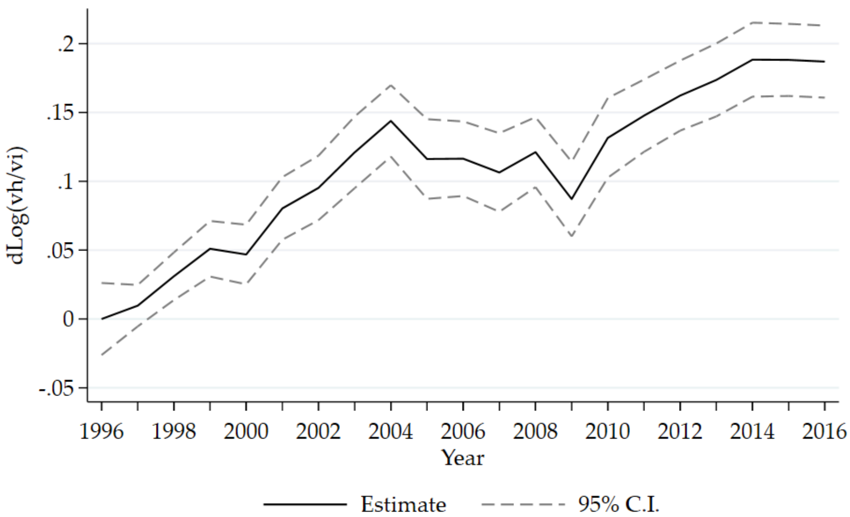
In particular, we would like to know if there have been important changes in spatial wage gaps between HQ and non-HQ establishments for a given occupation. In particular, we estimate the following regression:

$$\log\left(\frac{v_h}{v_i}\right)_t = \alpha_i + \delta_t + \varepsilon_{it}, \tag{4}$$

where v_i corresponds to the wage of managers at the establishment i and v_h to the wage of managers at its HQ h. Figure 3 shows the estimated δ_t. This figure shows that since 1996, there has been an average increase of around 19% in the wage of managers at firms' HQ relative to non-HQ establishments. Together with Figure 2, this trend also implies that workers at HQ are also becoming more expensive. Even though these are descriptive facts and do not constitute a causal relation, we consider this to be puzzling: why if managers at the HQ are becoming more expensive relative to non-HQ establishments, HQ establishments seem to be specializing in managerial activities? In the next section, we show that changes in relative wages across locations can be behind some of the changes in establishment specialization. This would also mean that when facing an exogenous shock to wages, firms with multiple establishments can substitute managers for workers not only within establishments, but also across establishments. This constitutes a novel mechanism that has not been explored in the literature.

Figure 3: Wage Ratio of Managers at HQ relative to non-HQ

Changes relative to 1996.



This figure shows the time fixed effects from a regression of the log ratio of the establishments' wages of managers at the HQ to managers at a non-HQ establishment on establishment and year fixed effects.

4 Model

Motivated by the facts, we develop a one-sector model in which firms decide the number of establishments, together with the locations and labor composition of each. The model builds on the literature on multinational firms (Helpman, 1984; Horstmann and Markusen, 1987; Markusen, 2002). In this literature, firms must decide whether to become multinational firms, given the existence of trade costs and differences in market access and labor costs. Moreover, the production technology involves the use of a firm-level public good, which gives rise to firm-level scale economies. However, there are decreasing returns to scale that create an incentive for establishing multiple plants. The complete mathematical derivations of our model can be found in section D form the Online Appendix.

Assume an economy with two locations $j \in \{o, s\}$, which can be thought of as the downtown of a city and its suburbs/hinterland. For simplicity, we assume that the location of the firm's HQ is fixed at $j = o$. Firms in this economy produce a single final good (Y) using land, managers (m) and production workers (L). Moreover, firms also use headquarters services (H), which can be thought of as the different services produced at the firms' HQ and that can be used by the other establishments. Recent research, such as Atalay et al. (2014), shows that these intangible inputs–and their transfer across establishments within firms–are crucial in explaining vertical integration and the formation of ME firms. These services include activities such as accounting, personnel, legal, monitoring, or problem-solving tasks, and tend to be manager-intensive activities (Aarland et al., 2007; Garicano, 2000).

Due to the existence of communication costs between establishments, not every unit of H produced at the HQ can be used in a particular establishment. Moreover, managers at the HQ can work producing either the final good or headquarters services, while managers at non-HQ establishments work only toward production of the final good. Workers at both the HQ and non-HQ establishments work only toward production of the final good. Final output also depends on a firm-specific productivity shock A_i.

Given these technological assumptions, the firm first decides whether to have one or two establishments–that is, whether to be a centralized or a fragmented firm– and the location of the potential second unit. In a second stage, the firm decides how much HQ services it produces. Finally, it chooses the number of managers and workers at each establishment. These decisions are driven by several forces. Wage differences between the HQ and other locations create an incentive for the firm to open new establishments and substitute workers out of the HQ to places where wages are relatively low. However, by opening new establishments, firms incur extra fixed costs of production and communication costs. They also could be missing out on some of the productivity advantages present at the HQ location. Finally, decreasing returns to scale in production causes centralized firms to be less profitable at the margin.

We assume that firms take the price of the final good (p), the local price of land (r_j, for $j \in \{o, s\}$), and local wages as given (w_j for the production workers and v_j for the managers, for $j \in \{o, s\}$). Given the nature of this problem, we solve it using backward induction. First, we present the labor composition problem at the establishment level, taking the number of establishments, their locations and the amount of HQ services as given. Based on this solution, we then analyze the problem of a firm choosing the optimal amount of HQ services. Finally, we study how the firm chooses between opening one or additional establishments and the location of potential additional establishments.

4.1 The Problem of the Establishment

An establishment i produces output using both types of labor, land and HQ services. Production of the final good in each establishment is Cobb-Douglas requiring production workers (L_i), a managerial bundle ($\mathbb{M}_{ij}$) and one unit of land. In particular, the managerial bundle is a function of local managers and the amount of headquarters services received by the establishment. The existence of communication costs between establishments causes attenuation of H; thus, establishments do not necessarily receive the full amount of H produced by the HQ. Communication costs take the form of an iceberg cost. In particular, when HQ produces 1 unit of HQ services, an establishment located in j receives $\tau_j \in [0, 1]$ units. We further assume that communication within location is free ($\tau_o = 1$), and we set $\tau_s = \tau$. Moreover, we assume that managers are more productive when they are at the HQ; thus, managerial input is augmented by $\mu_o > 1$ at the HQ.[9]

Summing up, an establishment i located in j solves the following maximization problem:

$$\max_{\{m_i, L_i\}} pY_{ij} - v_j m_i - w_j L_i - r_j \quad \text{s.t.} \quad Y_{ij} = AL_i^{\alpha}\mathbb{M}_{ij}^{\beta} = AL_i^{\alpha}\left[f(\mu_j m_i, \tau_j H)\right]^{\beta}, \tag{5}$$

where $\alpha, \beta, \in (0, 1)$ and $\alpha+\beta < 1$, $\mu_o > 1$ and $\mu_s = 1$, v_j is the wage of managers in location j, w_j the wage of production workers in location j and r_j denotes the price of land in j. The solution of this problem yields the demand for both inputs as functions of prices, firm productivity, HQ agglomeration economies, communication costs and HQ services. Moreover, we obtain the establishment's profit, which we denote by $\pi_i(\tau_j H)$.

4.2 The Problem of the Firm

Given the establishments' profit functions, the firm first chooses the total number of establishments and their respective locations. Second, the firm chooses the optimal amount of HQ services that maximizes the firm's profits. We assume that HQ services are produced using only managers from the HQ: $H = \mu_o m_h$, where m_h denotes the number of managers at the HQ producing H.[10]

We start by solving this last problem. Given the equation for H, this problem is equivalent to choosing the number of managers producing H that maximizes the sum of profits within the firm, net of the wage cost of these managers. Specifically,

$$\max_{\{m_h\}} \sum_{i\in\mathbb{E}} \pi_i(\tau_j \mu_o m_h) - v_o m_h, \tag{6}$$

where $\mathbb{E}$ represents the set containing all of the firm's establishments and j is the location of establishment i.

However, to derive the model's labor demand equations, we must first specify a functional form for the managerial bundle $\mathbb{M}_{ij}$, i.e., the relation between production managers and HQ services. Intuitively, if both types of managers were complements, establishments that receive a high amount of HQ services would need to hire more local managers–for example, the satellite establishment may need more managers in order to process the information it receives from the

[9] Figure A4 shows the structure of the model for a firm that has HQ in o and a establishment in s, respectively.
[10] This assumption can be relaxed to allow H to be produced using both types of labor. Doing so does not change the model's prediction, only makes them less stark.

HQ. On the other hand, if both types of managers were substitutes, establishments receiving a high amount of HQ services would need fewer local managers in order to produce the final output–for example, if some of the work done by local managers could be done by HQ managers at the same time for all of the establishments.

As we show in Section 6, our empirical estimates suggest a complementary relation between local and HQ managers. In particular, we find that the elasticity of substitution of managers across locations, with respect to a change in the corresponding relative wage, fades through distance. Moreover, this relation has become stronger over time. This prediction is consistent with a fixed-proportions production function for the managerial bundle of the form $\mathbb{M}_{ij} = \min\{\lambda\mu_j m_i, \theta\tau_j H\}$, where λ and θ are technology-determined constants.[11]

In section D.1 of the Online Appendix, we present the solution of the firm's problem both when it is a single-establishment firm (centralized) and when it has two establishments (fragmented solution). The solution of the centralized problem shows that single-establishment firms hire more of both types of workers when there are higher productivity advantages at the HQ, lower input prices, or a higher price for the final good, which can be thought of as a proxy for the demand faced by the firm.

More interestingly is the solution of the multi-establishment firms, which we summarize in four points. First, a reduction in communication costs ($\uparrow \tau$) leads to a increase in the demand for both types of workers in the satellite establishment, as well as an increase in the firm's profits. Second, higher wages lead to a decrease in factor demands and in the profitability of the firm. Third, since the production of the final good at any establishment requires HQ services, the demand for both types of workers in the satellite establishment depends not only on the price of workers and managers in that establishments, but also on the price of HQ managers. Moreover, an increase in the price of HQ managers leads to a decrease in the demand for both managers and workers at both establishments. Fourth, stronger decreasing returns to scale lead to lower profits and lower factor demand.

Finally, in a first stage, the firm chooses the number of establishments and their respective locations. Given the model's current setting, this decision is equivalent to choosing between having only one establishment, opening a second in o, or opening a second in s. Intuitively, a firm will open its satellite establishment in s if the marginal benefit of having that establishment is larger than the extra fixed cost of opening it. This marginal benefit depends on a trade-off between wage differences, communication costs and HQ agglomeration advantages. On the one hand, large wage differences between a firm's HQ and other locations create an incentive for the firm to open a new establishment in places where wages are relatively low. On the other hand, by opening new establishments firms incur extra communications cost and could miss out on the productivity advantages present at the HQ location.

[11] A Cobb-Douglas or a linear function would imply the opposite direction for the above prediction. In section D.4 of the Online Appendix, we present a version of the model using a Cobb-Douglas managerial bundle. Ideally, we would like to derive the model using a CES function. However, due to the non-linear nature of our model, it quickly becomes untractable.

4.3 Model Predictions

In this subsection, we present the main results of our model. In particular, we show how the firm's relative labor demands across and within establishments change when wages and communication costs change. The derivations and proofs of these results are shown in section D.1 of the Online Appendix. For ease of presentation, let us assume that the firm locates its second establishment in $j = s$.

Labor composition within Establishments

Start by analyzing the solution of the labor demand problem of a SE firm. The ratio of manager to workers for this firm is given by

$$\left(\frac{M_1}{L_1}\right)_{SE} = \frac{\beta}{\alpha}\frac{w_o}{v_o}. \tag{7}$$

This ratio depends only on the establishment's relative wage between managers and workers. Therefore, when a SE firm faces an exogenous shock that makes managers more expensive relative to workers, it can only respond by hiring fewer managers and/or more workers.

This is not the case for establishments that belong to a ME firm. First, consider the ratio of managers to workers at non-HQ establishments given by:

$$\left(\frac{M_2}{L_2}\right)_{ME} = \frac{\beta}{\alpha}\frac{w_s}{v_s}\left(1 + \frac{v_o}{v_s}\frac{\lambda}{\theta\tau\mu}\right)^{-1}. \tag{8}$$

Just as SE firms, this ratio depends negatively on the relative wages between managers and workers within the establishment. However, notice that it also depends negatively on the price of managers at the HQ relative to the establishment. This implies that an exogenous increase in the price of HQ managers raises the cost of the managerial inputs used at the non-HQ establishment, decreasing the demand for local managers. Finally, note that lower communication costs (τ) and higher HQ agglomeration economies magnify this effect.

Moreover, the ratio of manager to workers at the HQ is given by:

$$\left(\frac{M_1}{L_1}\right)_{ME} = \frac{\beta}{\alpha}\frac{w_o}{v_o}\left\{1 + \left[\left(\frac{\theta\tau}{\lambda}\right)^{\beta}\left(\frac{w_o}{w_s}\right)^{\alpha}\left(\frac{v_s}{v_o}\frac{\theta\tau\mu}{\lambda} + 1\right)^{\alpha-1}\right]^{\frac{1}{1-\alpha-\beta}}\right\}. \tag{9}$$

In addition to the standard labor demand channel, this ratio also depends on the wage gap of workers and managers between both locations, communication costs and productivity advantages at the HQ. This equation suggests that lower wages at a non-HQ establishment would lead to a higher manager-to-workers ratio at at the HQ. Moreover, lower communication costs magnify the effect that differences in the wages of managers and workers have on the HQ's managerial intensity. Finally, lower communication costs themselves have an ambiguous effect on managerial intensity at the HQ. On one hand, since non-HQ establishments receive more H, there will be an incentive to produce more HQ services and hire more managers at the HQ. On the other hand, if the establishment receives a higher quantity of HQ services, it also needs to hire more local managers, which would drive down total profits and the demand for H.

It is evident now that when facing an exogenous wage shock, ME firms have an extra margin of adjustment that SE firms do not have: They can substitute labor across locations. Moreover, firms have a third margin of adjustment given by the scale effects brought by the public nature of H. To see this more clearly, consider an exogenous shock that renders managers in the central location more expensive. Given that HQ managers are used in the production of both the final good and HQ services (used by all establishments), the direction of this effect is not straightforward. Using a Slutsky-type decomposition, the total effect of this shock on the manager-to-worker ratio at the HQ can be decomposed as the sum of four effects:

$$\begin{aligned}
\frac{\partial M_1/L_1}{\partial v_o} &= \frac{\partial (M_1/L_1)^c}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial v_o} \\
\frac{\partial M_1/L_1}{\partial v_o} &= \underbrace{\frac{\partial (M_1/L_1)^c}{\partial (v_o/w_o)} \cdot \frac{\partial (v_o/w_o)}{\partial v_o}}_{\text{Standard Subst. Effect}} + \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/w_o)} \cdot \frac{\partial (v_o/w_o)}{\partial v_o}}_{\text{Standard Scale Effect}} \\
&+ \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o}}_{\text{HQ Services / Firm Scale Effect}},
\end{aligned} \tag{10}$$

where the superscript c denotes the conditional labor demand functions that result from the firm's cost minimization problem, and Y_1 and Y_2 correspond to the total amount of output produced at HQ and non-HQ establishments, respectively. In Section D.3 from the Online Appendix, we show the details of this decomposition.

The first two terms on the right-hand side are the standard substitution and scale effects from a regular Slutsky-type equation. On one hand, if managers are more expensive, the establishment can substitute managers for workers (substitution effect). On the other hand, since the production of the final output requires both managers and workers, more expensive managers lead to a reduction in the establishment's output, which drives down the number of both types of workers at the HQ (scale effect). Both of these effects lead to a decrease in the ratio of managers to workers.

However, keeping the wage differences between managers and workers within the establishment constant, other effect arises. As HQ managers become more expensive, the firm will relocate both managers and workers to the satellite establishment. As this establishment becomes larger, the demand for HQ-services will also increase, hence increasing the demand for managers at the HQ and the ratio of managers to workers.

These two forces change the ratio of manager to workers at the HQ in different directions, but, it can be shown that, the latter effect dominates the former. Therefore, an exogenous shock that renders the price of managers more expensive at the central location can lead to an increase in the HQ's managerial intensity.Taking together the effects of an increase in the wage ratio v_o/v_s on the manager-to-workers ratio at HQ and non-HQ establishments, we can conclude that a wider managerial wage gap leads to a more manager-intensive HQ relative to non-HQ establishments. We formalize all these results in the next proposition. Formal proof of all propositions are included in Section D.1 from the Online Appendix.

Proposition 1. ***Managerial Intensity***

(a) Higher relative wages at the establishment's location ($\uparrow \frac{v}{w}$) lead to less manager-intensive establishments ($\downarrow \frac{M}{L}$). This holds for every establishment in the economy.

(b) For multi-establishment firms, a wider worker wage gap across locations ($\uparrow \frac{w_o}{w_s}$) leads to a more manager-intensive HQ relative to the non-HQ establishment ($\uparrow \frac{M_h/L_h}{M_i/L_i}$).

(c) For multi-establishment firms, a wider managerial wage gap across locations ($\uparrow \frac{v_o}{v_s}$) leads to a more manager-intensive HQ relative to the non-HQ establishment ($\uparrow \frac{M_h/L_h}{M_i/L_i}$). Lower communication costs ($\uparrow \tau$) magnify the effect.

We can also derive the elasticities of the HQ manager-to-worker ratio with respect to a change in the wage gap of either workers or managers across locations:

Corollary 1. *Keeping the within-establishment wage ratio ($\frac{v}{w}$) constant and defining $M_1 = m_1 + m_h$, the elasticities of the HQ ratio of managers to workers, with respect to changes in the wage gap of workers and managers across locations are given by:*

(a) For workers:

$$\frac{\partial \log(M_1/L_1)}{\partial \log(w_o/w_s)} = \frac{\alpha}{1-\alpha-\beta} \cdot \frac{v_o m_h}{v_o M_1} > 0.$$

(b) For managers:

$$\frac{\partial \log(M_1/L_1)}{\partial \log(v_o/v_s)} = \frac{1-\alpha}{1-\alpha-\beta} \cdot \frac{v_s/\lambda}{(v_s/\lambda)+(v_o/\theta\tau\mu)} \cdot \frac{v_o m_h}{v_o M_1} > 0.$$

Labor composition across Establishments

We also derive two equations that can help us understand the substitution patterns of workers and managers across establishments. Empirically, these two equations are useful for two reasons. First, since these equations yield relatively simple linear regressions, so their reduced-form estimates will help us test some of the model's predictions and, therefore, its validity. Second, the across-location elasticities of substitution implied by these equations provide important variation to identify the structural parameters of our model. In particular, we can write the ratio of workers and managers at the HQ relative to the non-HQ establishment as

$$\frac{L_1}{L_2} = \left[\left(\frac{w_o}{w_s}\right)^{\beta-1}\left(\frac{v_s}{v_o}\mu + \frac{\lambda}{\theta\tau}\right)^{\beta}\right]^{\frac{1}{1-\alpha-\beta}}, \tag{11}$$

$$\frac{M_1}{M_2} = \frac{\lambda}{\theta\tau\mu} + \frac{1}{\mu}\left[\left(\frac{w_o}{w_s}\right)^{-\alpha}\left(\frac{v_s}{v_o}\mu + \frac{\lambda}{\theta\tau}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha-\beta}}. \tag{12}$$

From these expressions, we highlight three effects. First, lower communication costs lead to a decrease in the ratios of workers and managers at the HQ relative to the other establishment. With lower communication costs, non-HQ establishments receive more services from the HQ and hire more managers and workers to produce more output. Second, higher HQ manager-specific productivity (μ) has a positive effect on the ratio of workers at the HQ relative to the non-HQ, but an ambiguous effect on the managerial ratio. On one hand, since managers are now more productive at the HQ, there is an incentive to increase the number of managers at the HQ relative to the other establishment. On the other hand, this rise in productivity also leads to a higher production of HQ services, thus driving up the demand for local managers at the non-HQ establishment.

Third, the share of workers and managers at the HQ depends negatively on the respective wage gap across locations. This result suggests that the size of the HQ relative to the non-HQ establishment, decreases when the wage gap across locations is wider. Even though this is an expected result, variation in these across-location elasticities of substitution will help us identify the model's structural parameters. From the these two equations, we calculate the respective across-location elasticities of substitution of workers and managers as

$$\begin{aligned} \frac{\partial \log(L_1/L_2)}{\partial \log(w_o/w_s)} &= \frac{\beta - 1}{1-\alpha-\beta} < 0, \\ \frac{\partial \log(M_1/M_2)}{\partial \log(v_o/v_s)} &= \frac{\alpha - 1}{1-\alpha-\beta} \cdot \frac{v_s/\lambda}{(v_s/\lambda) + (v_o/\theta\mu\tau)} \cdot \frac{v_o m_1}{v_o M_1} < 0, \end{aligned} \tag{13}$$

where $M_1 = m_1 + m_h$. While the elasticity of substitution of workers across establishments is constant, this is not the case for managers. The latter elasticity depends on two cost shares: (i) the cost share of non-HQ managers on the total managerial cost of the non-HQ establishment, and (ii) the cost share of production managers on the total managerial costs at the HQ. In particular, if production is not dependent on HQ services, the elasticity converges to a constant given by $\alpha - 1/1 - \alpha - \beta$. Moreover, as the use of HQ services increases, this elasticity converges to zero. Therefore, this elasticity implies that when firms are highly dependent on the within-firm public good, we would observe a low degree of substitution of managers across establishments for a given exogenous change in the wage gap of managers. Furthermore, if $\beta < \alpha$, the across-location elasticity of substitution for workers will always be more negative than the one for managers. These results can be summarized as:

Proposition 2. ***Substitution Across Establishments***

(a) A steeper wage gap for workers ($\uparrow \frac{w_o}{w_s}$) leads to a lower share of workers at the HQ ($\downarrow \frac{L_h}{L_i}$).

(b) A steeper wage gap for managers ($\uparrow \frac{v_o}{v_s}$) leads to a lower share of managers at the HQ ($\downarrow \frac{M_h}{M_i}$). Lower communication costs ($\uparrow \tau$) magnify the effects.

Finally, our model also provides us with a results on the firm's decision regarding its number of establishments:

Proposition 3. *The average number of establishments per firm increases with either lower* *communication costs, higher relative wages at the HQ (for either type of labor), higher productivity advantages at the HQ, higher firm-specific productivity, or higher differences in the price of land.*

5 Empirical Implementation

In addition to the testable theoretical implications, the model developed in the previous section provides a system of labor demand equations that can be estimated using establishment-level data. This estimation will allow us to decompose the changes in HQ managerial intensity that we observe in the data into the different mechanisms proposed by the model. In this section, we show how we implement the model empirically. Following the model, we use subindex i to denote an establishment and h to denote its HQ. Subindices j and k denote the establishment's and its HQ's location, respectively, while t represents year. Finally, s denotes the establishment's sector. For example, a variable such as M_{ijhst} would represent the number of managers in establishment i located in municipality j, with HQ h, in sector s at year t.

Due to the lack of detailed data on communication/fragmentation costs, we assume that communication costs between two locations depend negatively on the distance between the locations and an elasticity parameter that changes over time, c_t. Specifically,

$$\tau_{jk} \equiv dist_{jk}^{-c_t}. \tag{14}$$

Therefore, we will infer communication costs out of changes in the estimated parameter c_t. This parameter c_t captures the percentage change in communication costs given by a 1% change in the distance to HQ. We will refer to this parameter as the distance elasticity. Specifically, changes over time in the effect of distance (between establishments and their HQ) on labor demand decisions will be the main source of identifying variation for this parameter.

One important caveat of the following equations is that they are derived assuming that the firm has only two establishments. Therefore, they ignore the wider firm network and the effects that higher costs at the n^{th} establishment can have on the other establishments, as suggested by Gumpert et al. (2022). We do not think this is a first-order issue in this context for two reasons. First, around half of the ME firms in our sample have only two establishments. Therefore, even if our model only includes two establishments, it would be appropriate for around 90% of the firms in our sample. Second, the equations that describe the substitution of labor across locations are defined for each establishment-HQ pair. Therefore, we are still capturing a great deal of the information given by all of the non-HQ establishments within the firm.

5.1 Labor Composition within Establishments

In our model, we have three types of establishments: SE firms and HQ and non-HQ establishments from ME firms. For each of these, our model generates equations for the demand for managers relative to the demand for workers. Assume that the demand for managers relative to workers at each establishment i located in j, belonging to a firm with HQ h, from sector s at time t, is composed of three parts: (i) the total number of workers (L_{ijhst}) or managers (M_{ijhst}); (ii) a multiplier $e^{-\delta_t}$ that captures time characteristics that affect the ratio of managers to workers at every establishment in the country in a given year; and (iii) a multiplier $e^{-\epsilon_{ijhst}}$ that captures unobservable-time-varying factors that affect the establishment's relative demand for workers. In particular, define the relative labor demand in establishment i as $\frac{M_i}{L_i}e^{-\delta_t-\epsilon_{ijhst}}$ and assume that ϵ_{ijhst} is i.i.d. with mean zero.

With this structure in mind, we can apply total differentiation with respect to wages and communication costs to the three resulting equations and put them together as:

$$\begin{aligned} d\log\left(\frac{M}{L}\right)_{ijhst} &= -d\log\left(\frac{v}{w}\right)_{it} + \mathbf{1}_{\{HQ\}}\frac{\alpha(1-\omega_h)}{1-\alpha-\beta}d\log\left(\frac{w_h}{w_i}\right)_t \\ &+ \left\{\mathbf{1}_{\{HQ\}}\frac{(1-\alpha)(1-\omega_h)(1-\gamma_i)}{1-\alpha-\beta} - \mathbf{1}_{\{N\}}\gamma_i\right\} d\log\left(\frac{v_h}{v_i}\right)_t \\ &- \left\{\mathbf{1}_{\{HQ\}}\frac{\beta-(1-\alpha)(1-\gamma_i)}{1-\alpha-\beta}(1-\omega_h) + \mathbf{1}_{\{N\}}\gamma_i\right\}\sum_t \mathbf{1}_t\cdot[c_t \log dist_{ih}] \\ &+ d\delta_t + \zeta_{js} + d\epsilon_{ijhst}, \end{aligned} \tag{15}$$

where $\mathbf{1}_{\{HQ\}}$ and $\mathbf{1}_{\{N\}}$ are indicator variables that equal one if an establishment is the HQ or a non-HQ of a ME firm, respectively. Moreover, $\gamma_i = \frac{v_k/\theta\tau_{jk}\mu}{(v_j/\lambda)+(v_k/\theta\tau_{jk}\mu)}$ corresponds to the cost share of non-HQ managers on the total managerial costs of the non-HQ establishment i, and $\omega_h = \frac{v_k m_h^y}{v_k M_h}$ corresponds to the cost share of production managers on total managerial costs at the HQ. We recover these two objects from observed data on wages and managers in each establishment,[12] for which we use their lagged value to avoid endogeneity problems. We also control for pre-trends at the establishment's sector*commuting area level (ζ_{js}), which help us with identification, as they allow us to control for unobserved trends that affect the managerial intensity of an establishment that belongs to a particular sector and region.[13]

5.2 Labor Composition across Establishments

We also apply total differentiation with respect to wages and communication costs to the equations solving for the relative ratios of managers and workers across establishments. Then, we can use a similar set of assumptions as before to express the estimating equations as:

$$d\log\left(\frac{L_h}{L_i}\right)_{st} = \frac{\beta-1}{1-\alpha-\beta}d\log\left(\frac{w_h}{w_i}\right)_t - \frac{\beta(1-\gamma_i)}{1-\alpha-\beta}d\log\left(\frac{v_h}{v_i}\right)_t$$

[12] We define the number of managers in HQ services as those categorized as "Directors and Chief Executives." Then, we define the number of managers in the production of the final good at the HQ, m_h^y, as the difference between the total number of managers at the firm's HQ (M_h) and the total number of managers in HQ services.

[13] Since the manager-to-worker ratio at the HQ depends on the wage gap between the HQ and a non-HQ establishment, we use the firm's longest-lived and largest establishment from our data to construct these ratios for firms that have more than two establishments.

$$- \quad \frac{\beta\gamma_i}{1-\alpha-\beta}\sum_t \mathbf{1}_t \cdot [c_t \log dist_{ih}] + d\delta_t + \zeta_{js} + d\eta_{ijhst}, \tag{16}$$

$$\begin{aligned} d\log\left(\frac{M_h}{M_i}\right)_{st} &= \frac{(\alpha-1)(1-\gamma_i)\omega_h}{1-\alpha-\beta} d\log\left(\frac{v_h}{v_i}\right)_t - \frac{\alpha\omega_h}{1-\alpha-\beta} d\log\left(\frac{w_h}{w_i}\right)_t \\ &+ \left[\frac{(1-\alpha)\gamma_i\omega_h}{1-\alpha-\beta} - (1-\omega_h)\right]\sum_t \mathbf{1}_t \cdot [c_t \log dist_{ih}] + d\delta_t + \zeta_{js} + d\nu_{ijhst}, \end{aligned} \tag{17}$$

where η_{ijkst} and ν_{ijkst} are i.i.d. with mean zero.

The main concern for the proper identification of the model parameters is that the wages we observe for each firm are an equilibrium outcome of different conditions in the labor market. Therefore, identification of the parameters of the production function from our model requires a source of exogenous variation in the changes of the relative wages of managers and non-managers, within and across locations. In the following section, we present a framework through which we generate shocks on the labor supply faced by the establishments.

5.3 Endogeneity and Labor Supply

5.3.1 Identification Strategy

Given the simultaneity bias in the estimation of labor demand equations, we use local labor supply shocks to recover the parameters of interest. We follow the approach proposed by Card (2001), and used in Lewis (2011) and Baum-Snow et al. (2018), to build labor supply shocks using immigration share-shifters. We use these shocks as instruments for changes in the wages paid by the establishments to both managers and non-managers. The idea behind this type of instrument is that immigrants are more likely to live in municipalities with a relatively high number of immigrants from their country of origin, regardless of local labor market conditions.[14]

We believe this instrument to be a valid identification strategy in our setting for four reasons. First, the share of employment held by immigrants in Denmark has been increasing constantly since 1994.[15] Using this type of instrument, Foged and Peri (2016) show that municipalities with a relatively large exposure to immigration experienced significant changes in their supply of low-skilled labor. Second, there is relevant exogenous variation across countries of origin and time of arrival of the immigrants. For instance, Foged and Peri (2016) identify eight countries, including Bosnia, Vietnam and Somalia, that have had a relatively large number of international refugees migrating to Denmark since 1994. Third, due to differences in the accession dates of different countries within the European Union, there is also variation in terms of sector, type of workers and country of origin of people who could enter Denmark without restrictions (Fackler,

[14]Two important mechanisms can underpin this rationale. First, immigrants are more likely to face a relatively higher amenity value in municipalities with more people from their country of origin. Second, job referral and network effects are also more likely to be larger in these enclaves than in municipalities with a relatively low number of people from the immigrants' country of origin.

[15]The share of employment held by foreigners grew during the whole period between 1994 and 2006. It decreased less than 1% during the global recession, but started to increase again after 2012.

2018). Fourth, immigration policy in Denmark changes regularly. For example, Denmark implemented a dispersal policy between 1986 and 1998 with the goal of distributing refugees across municipalities in proportion to population size and available public housing (Damm, 2009). This policy generated national clusters of refugees that were independent of local labor market conditions.[16]

We start by building a standard immigration share-shifter for both managers and workers. Define IS^R_{ojt} and IS^F_{ojt} as the immigration shocks for type of worker o at the respective residence (R) and workplace (F) municipality j in year t. We construct our instrument in two steps. First, define the immigration shock at the residential municipality as

$$IS^R_{ojt} = \sum_c \frac{L_{ocjt^0}}{L_{oct^0}} \cdot \log Imm_{o,c,DK-j,t}, \tag{18}$$

where L_{ocjt^0} denotes the number of workers of occupation o from country of origin c in municipality of residence j in a base year t^0; L_{oct^0} denotes the total number of workers in Denmark in occupation o from country c in a base year t^0. Finally, $\log Imm_{o,c,DK-j,t}$ denotes the logarithm of the total stock of immigrants of occupation o from country of origin c in year t, leaving municipality j out of the calculation to avoid any endogeneity problems in the location of immigrants. We choose the period between 1986 and 1994 as our base period. During these years, the dispersal policy was at its full strength. Moreover, the number of immigrants started to increase systematically after 1994 (Foged and Peri, 2016). Thus, we keep 21 years of data (1996-2016) for our empirical exercises.

Since these immigrant enclaves are not necessarily located in the same municipalities as the establishments, immigration shocks at the residential municipalities might not be too relevant from the establishments' perspective as they may not shift labor supply curves in their municipality. In fact, due to differences in commuting between municipalities, a labor supply shock in one municipality might disperse geographically into other municipalities. For example, the labor supply that establishments experience in a municipality with high employment density, such as Aarhus, will be affected not only by the "residential" immigration shocks in Aarhus, but also by the shocks happening in nearby municipalities, such as Favrskov or Silkeborg.

Therefore, we augment the standard immigration share-shifter by mapping these shocks from municipalities of residence into workplace municipalities using commuting flows. In particular, we use historical commuting flows for Danish native workers in occupation o, π^D_{ojk,t^0}, in order to use a set of people different from the one we used for the immigration shocks. Note that we use occupation-specific commuting shares in order to capture possible differences in commuting costs between managers and workers. More specifically, our instrument is defined as

$$IS^F_{okt} = \sum_j IS^R_{ojt} \cdot \pi^D_{ojk,t^0}. \tag{19}$$

[16]The assignment of refugees to municipality was random conditional on their age, marital status, family size and nationality. This policy has been widely studied in the literature, starting with Damm (2009), and has been largely used for studying the impact of immigration on several worker-level outcomes (e.g., Foged and Peri (2016); Eckert et al. (2022)).

For our instrument to be valid, it must be that contemporaneous predicted immigration flows are not correlated with unobserved factors driving changes in firms' labor demand decisions. That is, we must assume that manager and non-manager immigrants arriving in Denmark before 1994 could not anticipate which firms were going to generate new jobs afterward. Specifically, consistent identification of the model's parameters requires that

$$E[\mathbf{dIS_t^F} \cdot d\varepsilon_t] = 0,$$

where $\mathbf{dIS_t^F} = \{dIS_{olt}^F\}_{\forall(o,l)}$, and $d\varepsilon_t = [d\epsilon, d\eta, d\nu]$ is the vector containing the residuals from the relative labor demand regressions (15), (16) and (17).

Finally, our identification strategy also relies heavily on the panel structure of our data. In all of our estimations, we include establishment and year fixed effects and pre-trends at the establishment's sector*commuting zone. Establishment fixed effects allow us to control for unobservable time-invariant establishment characteristics that might alter the establishments' relative labor demand decisions. Therefore, our identification comes from changes in relative labor demand decisions within and across locations, not from the levels. Moreover, year fixed effects control for outside factors in a given year that affect all of the firms equally. Finally, sector*commuting area trends allow us to control for unobserved trends that affect relative labor demand decisions in a particular sector-region pair.

5.3.2 First Stage Results

We start by estimating what can be considered reduced-form labor supply equations. Specifically, we estimate regressions of the establishment-level wage growth on the respective occupation-municipality labor supply shocks, as follows:

$$\begin{aligned} d\log v_{ijhst} &= a_1 dIS_{M,jt}^F + d\delta_t^M + \zeta_{js}^M + de_{M,ijhst}, \\ d\log w_{ijhst} &= b_1 dIS_{L,jt}^F + d\delta_t^L + \zeta_{js}^L + de_{L,ijhst}, \end{aligned} \tag{20}$$

where $\log v_{ijhst}$ and $\log w_{ijhst}$ represent, respectively, the logarithm of the wage of managers and workers in establishment i located in municipality j, from a firm with HQ h, in sector s and year t. Recall that these wages are the quality-adjusted wages we built in Section **??**.

Columns (1) and (2) from Table 1 present the results of these two equations. These results indicate that establishments located in municipalities with relatively large predicted immigration flows decreased the wages paid to both managers and non-managers, as expected. Workers living in municipalities that saw a relatively higher influx of immigrants will have a relatively tighter labor market, and might be willing to accept a job at a lower wage.[17] However, the standard

[17]This result differs from the estimates presented by Foged and Peri (2016) who find a positive effect of refugees on wages of less educated Danish. However, even if our immigration shock is motivated by theirs, our results are not easily comparable as our empirical specifications and sample are different. First, we estimate these regression using wage indices at the establishment level, while Foged and Peri (2016) use worker level wages. Second, we

errors and the F-tests from these regressions indicate that most of the identifying variation will come from labor supply shocks for non-managers. This is to be expected, since a large share of the immigrants arriving in Denmark are refugees from countries with relatively lower education levels and are not migrating as managers. Moreover, the location choice of managers (both immigrants and natives) is more likely to be affected by other types of consumption amenities, and not necessarily by enclave-type amenities (Couture and Handbury, 2020).

Table 1: Establishment-level Wages and Labor Supply Shocks

	$\log v_i$	$\log w_i$	$\log\left(\frac{v}{w}\right)_i$	$\log\left(\frac{w_h}{w_i}\right)$	$\log\left(\frac{v_h}{v_i}\right)$
Immigration Shocks	(1)	(2)	(3)	(4)	(5)
IS^F_{Mj}	-0.0004*				
	(0.0002)				
IS^F_{Lj}		-0.0007***			
		(0.0001)			
$IS^F_{Mj} - IS^F_{Lj}$			-0.0009***		
			(0.0001)		
$IS^F_{Lk} - IS^F_{Lj}$				0.0002***	
				(0.000)	
$IS^F_{Mk} - IS^F_{Mj}$					0.0014***
					(0.0002)
N			263,621		
F-Test	3.32	75.90	62.93	68.66	44.36

Notes: This table shows the results of regressions of establishment-level wages on the respective commuting-augmented immigration shock. As dependent variables, column (1) uses only managers, column (2) only workers, column (3) the ratio of managers to workers within establishment, column (4) the ratio of workers at the HQ relative to non-HQ, column (5) the ratio of managers at the HQ relative to non-HQ. j and k correspond to the location of the establishment and its HQ, respectively. Standard errors clustered by establishment's municipality. All regressions include establishment and year fixed effects and sector*commuting area pre-trends. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

In the context of our relative labor demand equations, we use different combinations of equations (20). These first stages can be thought of as relative labor supply equations complementing our relative labor demand system, as modeled by Baum-Snow et al. (2018). In particular, we use: (i) the immigration shock of managers relative to workers in the establishment's municipality as an instrument for changes in the establishment's relative wage between managers and workers; (ii) the immigration shock of workers at the HQ location relative to the shock at the establishment's municipality as an instrument for changes in the relative wage of workers at the HQ, relative to the satellite establishment; and, (iii) similarly for managers. These equations can be specified as:

$$d\log\left(\frac{v}{w}\right)_{ijhst} = b_R d(IS^F_{Mjt} - IS^F_{Ljt}) + d\delta^R_t + \zeta^R_{ls} + de_{1,ijhst},$$

$$d\log\left(\frac{w_h}{w_i}\right)_{st} = b_L d(IS^F_{Lkt} - IS^F_{Ljt}) + d\delta^L_t + \zeta^L_{ls} + de_{2,ijhst},$$

use immigration from all countries (not only seven), ten more years of data and the commuting augmentation.

$$d\log\left(\frac{v_h}{v_i}\right)_{st} \quad = \quad b_M d(IS^F_{Mkt} - IS^F_{Mjt}) + d\delta^M_t + \zeta^M_{ls} + de_{3,ijhst}, \tag{21}$$

where i denotes the establishment, h their HQ, and j and k their respective locations. We cluster standard errors at the establishment's-HQ's municipality pair level. Columns (3) to (5) from Table 1 presents the estimates of b_R, b_M and b_L from these equations. These three estimates are statistically significant. Moreover, the Kleibergen-Paap rk Wald F statistics are above 10 in all three cases. It is also possible to specify these regressions with different coefficients for each part of the relative immigration shock (e.g., managers vs. non-managers or HQ location vs. establishment location). Both specifications yield quite similar results.

5.4 Estimation Procedure

We start by estimating reduced-form regressions that will help us test some of the model's predictions and confirm the validity of our model. In particular, using the first stage equations from Table 1, we start by estimating the following relative labor demand equations using two-stage least squares (2SLS):

$$\begin{aligned}
d\log\left(\frac{M}{L}\right)_{ijhst} &= \psi_R d\log\left(\frac{v}{w}\right)_{it} + d\delta^R_t + \zeta^R_{js} + d\epsilon_{ijhst}, \\
d\log\left(\frac{L_h}{L_i}\right)_{st} &= \psi_L d\log\left(\frac{w_h}{w_i}\right)_t + d\delta^L_t + \zeta^L_{js} + d\eta_{ijhst}, \\
d\log\left(\frac{M_h}{M_i}\right)_{st} &= \psi_M d\log\left(\frac{v_h}{v_i}\right) + d\delta^M_t + \zeta^M_{js} + d\nu_{ijhst}.
\end{aligned} \tag{22}$$

In order to explore the heterogeneous effects of exogenous wage changes, we continue by including an interaction term between changes in relative wages and (i) the distance between the establishment and its HQ, and (ii) a binary variable that indicates whether a firm has its HQ in the Copenhagen metropolitan area as a measure of agglomeration.[18] Finally, we explore how the interaction between changes in relative wages and distance evolved over our sample period.

Nonetheless, our main goal is to structurally estimate the parameters of our model. Therefore, we jointly estimate equations (15), (16), (17), and their respective relative labor supply equations from (21) using feasible generalized simultaneous nonlinear least squares (FGNLS). This method allows the errors to be correlated across equations. Note that all of our structural equations have the following form:

$$dY = f(\phi, W)dX + d\varepsilon,$$

where $\phi = \{\alpha, \beta, \{c_t\}\}$, X corresponds to the vector of endogenous variables (wage ratios) and

[18] Copenhagen's commuting area is the country's main labor market area. Eckert et al. (2022) show that both wages and the returns to experience are significantly larger in Copenhagen compared with the rest of the country, which can be interpreted as evidence that a Copenhagen dummy variable can be a good proxy for agglomeration.

W corresponds to exogenous variables (the shares ω_h and γ_i). As shown in Cai et al. (2006) and discussed in Baum-Snow et al. (2018), after incorporating a first stage as $dX = b \cdot dZ + de$, with $E[dZ \cdot de] = 0$, in the system of equations, the parameter vector ϕ can be identified and consistently estimated. In all of the estimations, we allow the errors to be correlated over time and establishments within municipalities.[19]

6 Results

We begin this section by presenting reduced-form estimates of the relative labor demand equations. Specifically, we test the model's propositions estimating three reduced-form equations that describe labor substitution patterns within and across establishments within firms. These regressions are also useful to show the variation that identifies the structural parameters. Afterward, we present estimates of our structural parameters, with which we quantify the relative importance of each one of the proposed mechanisms in generating the observed increase in the ratio of managers to workers at the HQ, relative to non-HQ establishments.

6.1 Reduced-Form Estimates

Table 2 reports estimates of within-establishment wage elasticities. In particular, Column (1) presents the estimation of the "standard" relative labor demand equation. The coefficient in this column represents the average elasticity of substitution between managers and non-managers. The results indicate that if the wage of managers relative to workers increases by 1%, the managers-to-workers ratio would decrease by 2.4%. We have not found any other estimates of the elasticity of substitution between managers and workers in the literature. However, our estimate is line with the elasticity of substitution between high- and low- skilled workers found in Baum-Snow et al. (2018) for the US: 2.6 during the 1990s and 3.6 since 2000.

In Column (2), we separate this estimate into the three different types of establishments: SE firms and HQ and non-HQ establishments from ME firms. The results in this column suggest that on average, the elasticity of substitution is higher in firms with one establishment than in firms with multiple establishments. In particular, we find that the elasticity of substitution between managers and workers is -3 in SE firms. This elasticity of substitution is significantly larger than that experienced at non-HQ establishments of ME firms (-1.2). These results indicate that when there is an exogenous shock that affects the wage of managers relative to workers, substitution is stronger in SE firms than ME firms. This makes sense when we consider that ME can also substitute labor across establishments, while decisions of SE firms are more constrained.

In order to see whether this elasticity of substitution varies depending on the establishment's

[19] In order to improve the computational efficiency of the structural estimation, we first demeaned our variables to account for establishment and year fixed effects. Based on Correia (2018), we apply the Frisch-Waugh-Lovell theorem, together with the method of alternating projections, to account for establishment and year fixed effects. Afterwards, we demeaned the resulting variables within each sector-commuting area pair in order to account for the sector-region pre-trends.

distance from its HQ, we estimate the same regression from Column (1) including an interaction term between distance to the HQ and relative wages. Column (3) suggests that the elasticity of substitution between managers and non-managers decreases with the establishments' distance to its HQ. In principle, this result would contradict our model, which indicates that communication costs do not affect the elasticity of substitution between the two types of workers. However, as we showed in equation (32), an exogenous shock to the wage of managers affects the ratio of managers to workers through the standard labor demand channel and through a combination of scale effects that depend positively on communication costs. Consequently, the combination of these two effects could be biasing our elasticity of substitution. Our structural model will be useful in order to separately identify these two effects, since we do not have an appropriate instrument that would allow us to do so in a reduced-form analysis.

Finally, we want to see whether this elasticity varies depending on agglomeration economies at the HQ's location. Therefore, we estimate the same regression including an interaction term between relative wages and a dummy variable that equals 1 if the HQ is located in Copenhagen's commuting area. The estimate from Column (4) suggests that the elasticity of substitution between managers and non-managers does not vary for those firms with a HQ located in the capital's region.

Table 2: Relative Labor Demand Estimation
Within-Establishment Wage Elasticities

	$d\log\left(\frac{M}{L}\right)$			
	(1)	(2)	(3)	(4)
$d\log\left(\frac{v}{w}\right)$	-2.392***		-3.135***	-2.431***
	(0.4315)		(0.434)	(0.447)
x Single-Est		-2.945***		
		(0.399)		
x HQ Multi-Est		-1.871***		
		(0.585)		
x non HQ Multi-Est		-1.146***		
		(0.376)		
Log(Dist HQ)			-0.001	
			(0.000)	
$d\log\left(\frac{v}{w}\right) *$ Log(Dist HQ)			0.339***	
			(0.065)	
HQ in Copenhagen				0.003**
				(0.001)
$d\log\left(\frac{v}{w}\right)_{ijt} *$ HQ in CPH				0.119
				(0.250)
N	263,621			
Number of Establishments	29,224			

Notes: This table shows the results from different regressions of the establishment's manager to worker ratio at the left hand side $d\log\left(\frac{M}{L}\right)$ as a function of their relative wage $d\log\left(\frac{v}{w}\right)$, distance to their HQ and a variable that equals 1 if their HQ is located in Copenhagen. Standard errors clustered by establishment's municipality in parentheses. All regressions include establishment and year fixed effects, and sector*commuting area pre-trends. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

In Table 3, we explore how the allocation of workers across establishments responds to exogenous changes in the wage gap of each type of worker. Column (1) shows the elasticity of substitution for workers between establishments and their HQ. Column (4) shows the equivalent elasticity for managers. To the best of our knowledge, these are the first estimates of the elasticity of substitution across locations of ME firms. In particular, Column (1) indicates that if wages of workers at the HQ relative to a non-HQ establishment increase by 1%, the relative number of workers at the HQ decreases by 3.5% on average. For managers, the elasticity of substitution across locations is not statistically different from zero. These results indicate that on average, firms respond more to spatial differences in the wages of workers than to differences in managerial wages. This would be in line with the results of our model–in particular, equation (35), which suggests a larger elasticity of substitution for workers than for managers.

Our model also suggests that the elasticity of substitution of managers across establishments should vary with distance from the HQ. Therefore, we estimate the same regressions and include an interaction term between distance and the respective wage ratio. In Columns (2) and (5), we present the results for workers and managers, respectively. Both columns indicate that the elasticities of substitution across HQ and non-HQ establishments decrease with the distance between the two establishments; although for non-managers this parameter is imprecisely estimated. Specifically, the results indicate that firms respond more to changes in the wage gap from closer establishments relative to farther ones. For example, consider a firm with two non-HQ establishments that experiences a similar exogenous reduction in the wages of workers (relative to the HQ) at both locations. This firm would rather increase the relative size of the closest establishment than the size of the more distant one. This could be caused by the ease of monitoring a nearby establishment, or because it is easier to transfer employees to closer establishments than it is to transfer them to a more distant workplace, given the existence of firing, hiring, and moving costs.

Consider again the coefficients from Column (2). Our results suggest that the elasticity of substitution between workers across establishments is 5.2 when an establishment is located in a municipality adjacent to its HQ. Moreover, the median establishment, which is located 97 km away from its HQ (according to Table A1), will experience an elasticity of substitution of 3.45. Similarly for managers, the coefficients from Column (5) suggest that the across-establishment elasticity of substitution for managers would go from 4.6 at a municipality adjacent to their HQ, to 0 at 97 km away. Moreover, the across-establishment elasticity for managers is only significantly smaller than zero for distances below 30 km, while the one for workers becomes statistically zero at 250 km.

Some of the results from Tables 2 and 3 would be compromised if our immigration instruments were correlated with the distance between the establishment and its HQ. This would be a problem, since our instrumental variable would be acting as a shock for both relative wages and distance. Thus, our instrument would not satisfy the exogeneity condition in the regressions from these tables. Therefore, we look at the correlations between distance from establishments to their HQ and the instruments, which are not statistically different from zero. We present these results in Table A4 in the Appendix. The previous correlations could suggest

Table 3: Relative Labor Demand Estimation
Across-establishment Wage Elasticities

	$\log\left(\frac{L_h}{L_i}\right)$			$\log\left(\frac{M_h}{M_i}\right)$		
	(1)	(2)	(3)	(4)	(5)	(6)
$d\log\left(\frac{wage_h}{wage_i}\right)$	-3.544***	-5.256***	-5.126***	-0.201	-4.626***	-1.896***
	(0.8152)	(1.617)	(1.717)	(0.3990)	(0.711)	(0.437)
Log(Dist HQ)		0.001			0.003***	
		(0.001)			(0.001)	
$d\log\left(\frac{wage_h}{wage_i}\right) *$ Log(Dist HQ)		0.394			1.008***	
		(0.333)			(0.144)	
HQ in CPH			0.003			0.011***
			(0.003)			(0.002)
$d\log\left(\frac{wage_h}{wage_i}\right) *$ HQ in CPH			1.873			1.990***
			(1.730)			(0.473)
N	263,621					
Number of Establishments	29,224					

Notes: This table shows the results from different regressions of the ratio of workers or managers at the HQ, relative to non-HQ establishments at the left hand side as a function of their respective relative wage, distance to their HQ and a variable that equals 1 if their HQ is located in Copenhagen. The wage ratio in Columns (1)-(3) corresponds to the worker wage ratio across locations $d\log\left(\frac{w_h}{w_i}\right)$. In Columns (4)-(6), it corresponds to the manager wage ratio across locations $d\log\left(\frac{v_h}{v_i}\right)$. Standard errors clustered by establishment's municipality. All regressions include establishment and year fixed effects and sector*commuting area pre-trends. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

that the labor supply shocks we consider are large enough to cause changes in the firms' labor demand decisions, but too small to influence the opening/closure of an establishment, since these extensive margin decisions involve large fixed costs.

Finally, in Columns (3) and (6), we include an interaction between wage gaps and the HQ-at-Copenhagen dummy in order to see whether these elasticities are affected by agglomeration economies at the HQ location. The point estimates suggest that the across-establishment elasticities of substitution are lower for those firms with HQ located in Copenhagen's labor market area. Although the estimates are similar in magnitude for both types of workers, it is only statistically different from zero for managers. Estimates in column (6) suggest that the elasticity of substitution of managers across locations is zero for firms with a HQ in Copenhagen. This result could suggest an important role for manager-biased agglomeration economies. More precisely, when firms with HQ in Copenhagen experience an exogenous shock that increases the wage of HQ managers, they might not want to substitute managers out of the HQ, perhaps because there are important productivity advantages of having them in that location. Unfortunately, as can be seen in Table A4, the correlations between the Copenhagen dummy and the instruments are not statistically different from zero. As we do not have an identification strategy that would let us estimate the causal effect of agglomeration economies, the estimates for these interactions should not be interpreted as causal.

We are also interested in knowing how the coefficients from the interactions between relative wages and distance to the HQ have changed over time, since this variation will identify the distance elasticity parameters $\{c_t\}$ in our structural estimation. In Table 4 we present the results from the within- and across-establishment wage elasticities, allowing the interaction term to vary over four periods: 1996-2000, 2001-2005, 2006-2010, and 2011-2016. Estimates from Column (1) suggest that the effect of distance from the HQ on the elasticity of substitution between managers and workers has been fading over time. Results from Columns (2) and (3) indicate that over time, ME firms seem to be less willing to substitute workers or managers between HQ and non-HQ establishments that are relatively far from the HQ.

Table 4: Relative Labor Demand Estimation and Distance*Time

	$d\log\left(\frac{M}{L}\right)$	$d\log\left(\frac{L_h}{L_i}\right)$	$d\log\left(\frac{M_h}{M_i}\right)$
	(1)	(2)	(3)
$d\log\left(\frac{v}{w}\right)$	-3.291***		
	(0.443)		
$d\log\left(\frac{w_h}{w_i}\right)$		-5.429***	
		(1.612)	
$d\log\left(\frac{v_h}{v_i}\right)$			-4.571***
			(0.675)
Dist. to HQ	-0.002***	0.001	0.002***
	(0.000)	(0.001)	(0.001)
$d\log(RelW)$ *Dist. to HQ*(1996-2000)	0.692***	-0.311	0.488***
	(0.116)	(0.381)	(0.167)
$d\log(RelW)$ *Dist. to HQ*(2001-2005)	0.531***	-0.233	0.720***
	(0.123)	(0.391)	(0.150)
$d\log(RelW)$ *Dist. to HQ*(2006-2010)	0.237***	1.157***	1.397***
	(0.051)	(0.377)	(0.147)
$d\log(RelW)$ *Dist. to HQ*(2011-2016)	-0.310*	0.875**	1.345***
	(0.171)	(0.384)	(0.170)
N		263,621	
Number of Establishments		29,224	

Notes: This table shows the results from regressions on within- and across-establishments relative labor demand on their respective relative wage ($d\log(RelW)$), distance to the establishment's HQ and their time-varying interaction. Standard clustered by establishment's municipality in parentheses; j and k denote the location of the establishment and its HQ. All regressions include establishment and year fixed effects and sector*commuting area pre-trends. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

The positive interaction between wage changes and distance, and the fact that it has become larger over time, can be interpreted as evidence supporting a complementary relation between local managers and lower communication costs. In terms of our model, this relation validates the choice of a fixed-proportions production function for the managerial bundle, instead of a Cobb-

Douglas or a perfect substitute function. These two specifications would imply a nonpositive coefficient for the interaction term. These results also imply that given an exogenous wage shock that makes two satellite locations cheaper, firms would be more willing to shift workers to those locations subject to lower communication costs. In our model, these locations receive more HQ services and thus they need more local managers. This result contrasts with recent research by Gumpert et al. (2022), who find that middle-managers at non-HQ establishments serve as a substitute for CEO time. Nonetheless, our result is in line with Bresnahan et al. (2002) and Fort (2017), who find evidence of complementarities between technology and worker skill. In particular, Fort finds that investments in communication technology lower fragmentation costs, but these effects are disproportionately higher for domestic than for foreign sourcing.

6.2 Estimates of the Structural Model

This subsection presents the estimates of α, β and $\{c_t\}$ that result from estimating equations (15), (16) and (17). Using these parameters, we use equation (15) to back out the relative importance of each proposed mechanisms in generating the observed increase in the managers-to-workers ratio at HQ relative to non-HQ establishments. Table 5 reports estimates of the production function parameters using counts of workers in Column (1).[20]

We estimate α to be 0.414 and β to be 0.376. These estimates suggest strong evidence of decreasing returns to scale, since $\hat{\alpha} + \hat{\beta}$ is significantly less than 1 at the 99% confidence level.Column (1) also reports the estimates of the elasticity of communication cost with respect to the distance between establishments and their HQ relative to 1996-1997. Our results suggest that the distance elasticity has been increasing since 2004. In light of our model, these estimates would imply an increase in communication costs between 1996 and 2016. Several factors could be behind these changes. First, even though communication costs have decreased in this period, the information firms need to collect and communicate could be becoming more complex. For example, modern firms collect a greater amount of data and are more dependent on technology than they were before. Moreover, technology can also cause an increase in the division of labor (Tian, 2017). This means that the effective cost of communication could be increasing, as HQ services could now be more complex and specialized. There is a need for further research exploring the role of knowledge and technology within firms.

Second, recall that these parameters are identified from variation across distance between establishments and time similar to the estimates from Table 4, where we show that the relation between relative wages and labor demand across location has been increasing. These increasing coefficients can be driving the increasing distance elasticities. Moreover, other factors, such as changes in transportation infrastructure, could also be affecting these changes. For example, the opening of the Great Belt Bridge (in 1998) and the Oresund Bridge (in 2000), which connect the Copenhagen area with the rest of Denmark and Sweden, respectively, could have had important

[20] Table A5 shows the estimated parameters from the first stage. Note that we cannot recover the technological parameters λ and θ. However, we do not need these parameters in order to perform the decomposition exercises.

effects on Denmark's economic geography, by changing the internal organization of firms.[21]

In Column (2), we report estimates of the production function parameters using efficiency units of labor; both set of parameters are quite similar. As a robustness check, we estimated our model including changes over time of the HQ-manager productivity parameter μ. This specification takes into account HQ time-variant characteristics that would render managers at the HQ more productive over time. We include this specification in Table A6. The parameter estimates do not change significantly from those in Table 5.

Table 5: Parameter Estimates

Parameter	Description	Counts	Eff. Units
α	Labor Share–Cobb-Douglas	0.414***	0.358***
		(0.048)	(0.054)
β	Managerial Share–Cobb-Douglas	0.376***	0.433***
		(0.039)	(0.044)
c_1	Distance Elasticity 1996-97	0.004***	0.003***
		(0.001)	(0.001)
c_2	Distance Elasticity 1998-99 (Rel. c_1)	-0.002	-0.003*
		(0.001)	(0.001)
c_3	Distance Elasticity 2000-01 (Rel. c_1)	-0.001	-0.001
		(0.002)	(0.002)
c_4	Distance Elasticity 2002-03 (Rel. c_1)	-0.001	-0.002
		(0.003)	(0.003)
c_5	Distance Elasticity 2004-05 (Rel. c_1)	0.007**	0.005*
		(0.003)	(0.003)
c_6	Distance Elasticity 2006-07 (Rel. c_1)	0.012***	0.009***
		(0.003)	(0.003)
c_7	Distance Elasticity 2008-09 (Rel. c_1)	0.026***	0.023***
		(0.004)	(0.004)
c_8	Distance Elasticity 2010-11 (Rel. c_1)	0.031***	0.026***
		(0.005)	(0.005)
c_9	Distance Elasticity 2012-13 (Rel. c_1)	0.034***	0.028***
		(0.005)	(0.005)
c_{10}	Distance Elasticity 2014-15 (Rel. c_1)	0.036***	0.030***
		(0.006)	(0.006)

Notes: This table shows the parameter estimates and standard errors from two specifications of the structural model. Exact estimation equations can be found in Section 5. Standard errors clustered by establishment's municipality in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

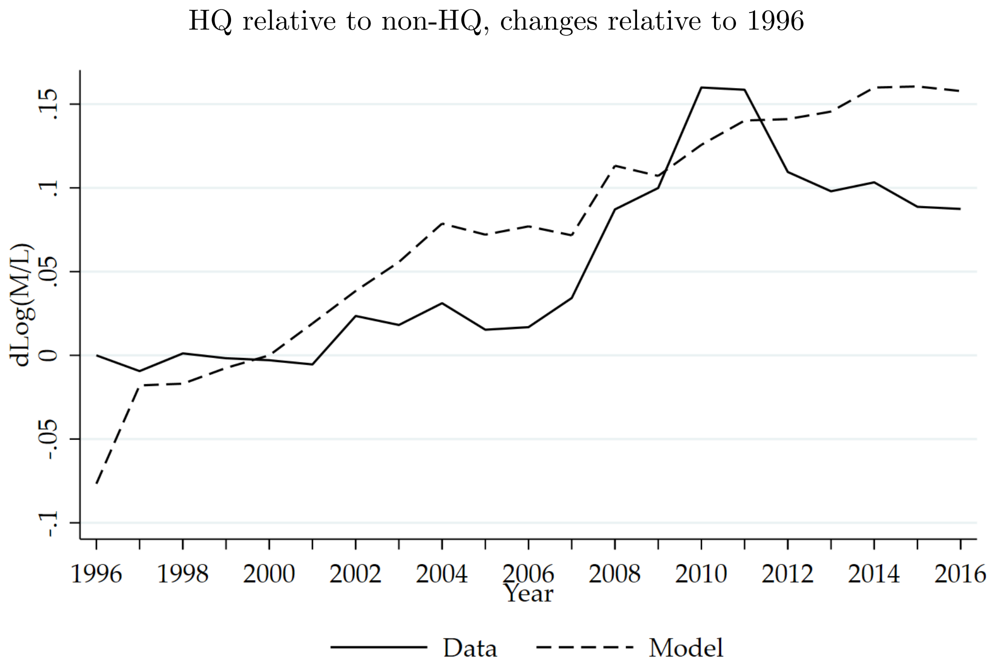
6.3 Decomposition of the Growth in HQ Managerial Intensity

Using the estimated parameters from Table 5, we investigate the extent to which changes in within-establishment wage differences, across-establishment wage gaps and distance elasticity can explain the increase in the manager-to-workers ratio at HQ, relative to non-HQ establishments, documented in Section 3.

[21] There is little research on the impact of these bridges on different economic outcomes. Two important exceptions are Mulalic et al. (2014) and Bütikofer et al. (2022).

In particular, we build each of the components from equation (15). The sum of these components gives us the total change in the manager-to-workers ratio predicted by our model. We regress this predicted ratio on year and year*HQ fixed effects, analogous to what we did in equation (2). The coefficients estimated by the interaction between year and HQ dummies gives us the change (relative to 1996) in the predicted manager-to-workers ratio at the HQ relative to non-HQ. In Figure 4, we plot this series together with the series from Figure 1. The figure shows that our model does a good job predicting the observed changes in the manager-to-workers ratio, in particular for the period between 1996 and 2009. However, the model fails to capture the cycle between 2010 and 2012, during which the Danish economy was hardly hit by the financial crisis. Since these periods can be associated with a state of disequilibrium, we are not greatly concerned about not matching this latter period.

Figure 4: Model Fit–Changes in Manager-to-Workers Ratio



Both lines from this figure corresponds to the year*HQ fixed effects of a regression on a manager-to-worker ratio at HQ, relative to non-HQ establishments, on establishment, year and year*HQ fixed effects. The solid line corresponds to the ratio observed in the data and presented in Figure 1. The dashed line corresponds to the ratio predicted by the model using the estimated parameters from Table 5.

In addition, we estimate a regression on each of the components from equation (15) on year and year*HQ fixed effects. The coefficients estimated by the interaction between year and HQ dummies in each regression gives the contribution of each component to the yearly change in the manager-to-workers ratio at the HQ relative to non-HQ establishment. Table 6 shows the results of the decomposition. The results of the decomposition are striking. Column (1) shows that the whole change in the increase in managerial intensity at the HQ cannot be explained by changes in the relative wages of managers to workers within establishments. In fact, this "standard" labor demand channel accounts for a -5% of the total change. Changes in distance elasticity explain around 40% of the total change.

Our results suggest that the largest contribution to the total change comes from increases in the managerial wage gap, which contributed 66%. Recall that in our model, this change comes from the interplay between two effects. First, a substitution effect: if managers are more expensive, the establishment can substitute managers for workers which can lead to a decrease in the ratio of managers to workers. Second, a HQ-services (firm-scale) effect: as managers become more expensive at HQ, the firm will relocate both managers and workers to the satellite establishment, causing an increase in its size and in the demand for HQ-services, thus for managers at the HQ. The two effects combined suggest that an exogenous shock that renders managers more expensive at the HQ location relative to managers at non-HQ locations, can lead to an increase in the relative managerial intensity of the HQ. Finally, as a robustness check, we allow for the HQ manager-enhancing productivity to change over time in Column (2). The main results remain unchanged.

Table 6: Decomposition of the growth of M/L

	1996-2006	
Changes in:	(1)	(2)
Within Est. Wage Diff - $d\log(v/w)$	-4.8%	-4.7%
Mgrial Wage Gap - $d\log(v_h/v_i)$	66.1%	64.2%
Worker Wage Gap - $d\log(w_h/w_i)$	0.1%	0.1%
Distance Elasticity	38.6%	44.5%
HQ productivity advantage		-4.1%
Total Change	100%	100%

Notes: This table shows the results of a decomposition of the growth of the manager-to-workers ratio at the HQ relative to non-HQs, relative to 1996, into the different components suggested by equation (15). Components were calculated using the parameters from Table 5. Each component was regressed on year and year*HQ fixed effects.

6.4 Number of Establishments

In Section E from the Online Appendix, we present descriptive evidence and numerical simulations to understand how changes in wages across locations, communication costs and agglomeration economies affect the firm's number of establishments and their location. Our estimates suggest that lower relative wages in a municipality offset the negative impact of the distance to the firm's HQ. Thus, a firm might be willing to open an establishment farther from its HQ if it offers a big enough cost advantage. Moreover, this relation has become stronger over time. This empirical evidence is supported by our numerical simulations. In particular, we find that changes in the wage gaps across locations lead to more firm fragmentation, and this effect strengthens as communication costs fall. In addition, we find that lower communication costs allow firms experiencing moderate levels of agglomeration economies to fragment and open a second establishment where both land and labor are relatively cheap. These results are in line with the empirical results from Acosta and Lyngemark (2021).

7 Conclusions

Over the last decades, dramatic changes have occurred in the internal spatial organization of firms along two dimensions. First, headquarters (HQ) establishments have become more manager intensive relative to satellite establishments, despite a significant increase in managerial wages at HQ locations. Second, firms have become more fragmented over time, opening more establishments and locating them farther from the firm's HQ. In this paper, we study, theoretically and empirically, how environmental changes faced by firms affect their spatial organization. To study these issues, we first developed a model of a multi-establishment firm to study the effect that changes in relative wages within and across locations, communication costs and agglomeration economies have on the establishments' labor composition and on firm fragmentation.

Using Danish administrative data, together with commuting-augmented immigration shocks as the source of identifying variation for changes in the relative supply of workers and managers within and across municipalities, we test the predictions of the model and estimate its structural parameters. We find that firms are more likely to substitute workers than managers out of their headquarters into satellite establishments. Moreover, this across-establishment elasticity of substitution is higher for closer than for distant locations and for single-establishment firms.

Finally, our decomposition indicates that increases in the wage gap of managers between HQ and non-HQ establishments accounts for around 66% of the increase in the managerial intensity at HQ establishments. This can be explained by the associated increasing demand for manager-intensive headquarter services, such as legal and accounting tasks, that must be used by satellite establishments as they become larger. Furthermore, changes in communication costs account for 39% of the change. To the best of our knowledge, this is the first paper to specify and estimate a structural model of the internal spatial organization of firms.

Given (i) the increasing importance of multi-establishment firms in the aggregate economy, together with (ii) the increasing patterns of firm fragmentation found in recent literature and (iii) the fact that headquarters are disproportionally located in large urban areas, these within-firm specialization patterns could significantly affect the aggregate economic geography of a country. Future research should focus on understanding the effects that changes in the internal spatial organization of firms have on the economic geography of a country. Future research should also focus on the impact that lower communication–and not only transportation–costs have on firms' spatial decisions.

References

Aarland, K., Davis, J., Henderson, J. V., and Ono, Y. (2007). Spatial organization of firms: the decision to split production and administration. *RAND Journal of Economics*, 38(2):480–494.

Acosta, C. and Lyngemark, D. H. (2021). The Internal Spatial Organization of Firms: Evidence from Denmark. *Journal of Urban Economics*, 124:103366.

Antras, P., Garicano, L., and Rossi-Hansberg, E. (2006). Organizing offshoring: Middle managers and communication costs. Technical report, National Bureau of Economic Research.

Antràs, P. and Yeaple, S. R. (2014). Multinational firms and the structure of international trade. In Gopinath, G., Helpman, E., and Rogoff, K., editors, *Handbook of International Economics*, volume 4, chapter 2, pages 55–130. Elsevier.

Antràs, P., Garicano, L., and Rossi-Hansberg, E. (2006). Offshoring in a Knowledge Economy. *The Quarterly Journal of Economics*, 121(1):31–77.

Atalay, E., Hortaçsu, A., and Syverson, C. (2014). Vertical integration and input flows. *American Economic Review*, 104(4):1120–48.

Bartelme, D. and Ziv, O. (2017). The internal geography of firms. Work in Progress.

Baum-Snow, N., Freedman, M., and Pavan, R. (2018). Why has urban inequality increased? *American Economic Journal: Applied Economics*, 10(4):1–42.

Becker, G. S. and Murphy, K. M. (1992). The division of labor, coordination costs, and knowledge. *The Quarterly Journal of Economics*, 107(4):1137–1160.

Behrens, K. and Sharunova, V. (2015). Inter-and intra-firm linkages: Evidence from microgeographic location patterns.

Bloom, N., Garicano, L., Sadun, R., and Reenen, J. V. (2014). The Distinct Effects of Information Technology and Communication Technology on Firm Organization. *Management Science*, 60(12):2859–2885.

Bresnahan, T. F., Brynjolfsson, E., and Hitt, L. M. (2002). Information technology, workplace organization, and the demand for skilled labor: Firm-level evidence. *The Quarterly Journal of Economics*, 117(1):339–376.

Bütikofer, A., Løken, K. V., and Willén, A. (2022). Building bridges and widening gaps. *Review of Economics and Statistics*, pages 1–44.

Cai, Z., Das, M., Xiong, H., and Wu, X. (2006). Functional coefficient instrumental variables models. *Journal of Econometrics*, 133(1):207–241.

Card, D. (2001). Immigrant inflows, native outflows, and the local labor market impacts of higher immigration. *Journal of Labor Economics*, 19(1):22–64.

Cestone, G., Fumagalli, C., Kramarz, F., and Pica, G. (2022). Exploiting growth opportunities: The role of internal labor markets. *The Review of Economic Studies*.

Chandler, A. D. (1969). *Strategy and Structure: Chapters in the History of the American Industrial Enterprise*. MIT Press.

Chandler, A. D. (1991). The functions of the HQ unit in the multibusiness firm. *Strategic Management Journal*, 12(S2):31–50.

Charnoz, P., Lelarge, C., and Trevien, C. (2018). Communication Costs and the Internal Organisation of Multi-plant Businesses: Evidence from the Impact of the French High-speed Rail. *The Economic Journal*, 128(610):949–994.

Collis, D., Young, D., and Goold, M. (2007). The size, structure, and performance of corporate headquarters. *Strategic Management Journal*, 28(4):383–405.

Collis, D., Young, D., and Goold, M. (2012). The size and composition of corporate headquarters in multinational companies: empirical evidence. *Journal of International Management*, 18(3):260–275.

Combes, P.-P., Duranton, G., and Gobillon, L. (2008). Spatial wage disparities: Sorting matters! *Journal of Urban Economics*, 63(2):723–742.

Combes, P.-P., Duranton, G., Gobillon, L., Puga, D., and Roux, S. (2012). The productivity advantages of large cities: Distinguishing agglomeration from firm selection. *Econometrica*, 80(6):2543–2594.

Correia, S. (2018). Reghdfe: Stata module to perform linear or instrumental-variable regression absorbing any number of high-dimensional fixed effects.

Couture, V. and Handbury, J. (2020). Urban revival in america. *Journal of Urban Economics*, 119:103267.

Damm, A. P. (2009). Determinants of recent immigrants' location choices: quasi-experimental evidence. *Journal of Population Economics*, 22(1):145–174.

Davis, J. C. and Henderson, J. V. (2008). The agglomeration of headquarters. *Regional Science and Urban Economics*, 38(5):445–460.

De La Roca, J. and Puga, D. (2017). Learning by working in big cities. *The Review of Economic Studies*, 84(1):106–142.

Diamond, R. (2016). The determinants and welfare implications of us workers' diverging location choices by skill: 1980-2000. *American Economic Review*, 106(3):479–524.

Duranton, G. and Puga, D. (2005). From sectoral to functional urban specialisation. *Journal of Urban Economics*, 57(2):343–370.

Eckert, F. (2019). Growing Apart: Tradable Services and the Fragmentation of the U.S. Economy.

Eckert, F., Hejlesen, M., and Walsh, C. (2022). The return to big-city experience: Evidence from refugees in denmark. *Journal of Urban Economics*, page 103454.

Fackler, T. (2018). *The effects of migration, competition, and patents on innovation.* PhD thesis, Ludwig-Maximilians-Universität München.

Fligstein, N. (1985). The spread of the multidivisional form among large firms, 1919-1979. *American Sociological Review*, pages 377–391.

Foged, M. and Peri, G. (2016). Immigrants' effect on native workers: New analysis on longitudinal data. *American Economic Journal: Applied Economics*, 8(2):1–34.

Fort, T. C. (2017). Technology and production fragmentation: Domestic versus foreign sourcing. *The Review of Economic Studies*, 84(2):650–687.

Garicano, L. (2000). Hierarchies and the Organization of Knowledge in Production. *Journal of Political Economy*, 108(5):874–904.

Gaubert, C. (2018). Firm sorting and agglomeration. *American Economic Review*, 108(11):3117–53.

Giroud, X. (2013). Proximity and investment: Evidence from plant-level data. *The Quarterly Journal of Economics*, 128(2):861–915.

Giroud, X. and Mueller, H. M. (2019). Firms' internal networks and local economic shocks. *American Economic Review*, 109(10):3617–49.

Gokan, T., Kichko, S., and Thisse, J.-F. (2019). How do trade and communication costs shape the spatial organization of firms? *Journal of Urban Economics*, 113:103191.

Gumpert, A., Steimer, H., and Antoni, M. (2022). Firm Organization with Multiple Establishments. *The Quarterly Journal of Economics*, 137(2):1091–1138.

Helpman, E. (1984). A simple theory of international trade with multinational corporations. *Journal of Political Economy*, 92(3):451–471.

Henderson, J. V. and Ono, Y. (2008). Where do manufacturing firms locate their headquarters? *Journal of Urban Economics*, 63(2):431–450.

Horstmann, I. J. and Markusen, J. R. (1987). Strategic investments and the development of multinationals. *International Economic Review*, pages 109–121.

Jiang, X. (2022). Information and Communication Technology and Firm Geographic Expansion.

Kim, S. (1999). The rise of multiunit firms in us manufacturing. *Explorations in Economic History*, 36(4):360–386.

Laamanen, T., Simula, T., and Torstila, S. (2012). Cross-border relocations of headquarters in europe. *Journal of International Business Studies*, 43(2):187–210.

Lewis, E. (2011). Immigration, skill mix, and capital skill complementarity. *The Quarterly Journal of Economics*, 126(2):1029–1069.

Markusen, J. (2002). *Multinational Firms and the Theory of International Trade*, volume 1. The MIT Press, First edition.

Menz, M., Kunisch, S., and Collis, D. J. (2015). The corporate headquarters in the contemporary corporation: Advancing a multimarket firm perspective. *The Academy of Management Annals*, 9(1):633–714.

Moretti, E. (2004). Human capital externalities in cities. In *Handbook of Regional and Urban Economics*, volume 4, pages 2243–2291. Elsevier.

Mulalic, I., Van Ommeren, J. N., and Pilegaard, N. (2014). Wages and commuting: Quasi-natural experiments' evidence from firms that relocate. *The Economic Journal*, 124(579):1086–1105.

Nielsen, B. (2005). Udviklingen i pendlingen siden år 2000. *Arbejdsnotat, Skov- og Naturstyrelsen.*

Oberfield, E., Rossi-Hansberg, E., Sarte, P.-D., and Trachter, N. (2020). Plants in space. Technical report, National Bureau of Economic Research.

Ota, M. and Fujita, M. (1993). Communication technologies and spatial organization of multi-unit firms in metropolitan areas. *Regional Science and Urban Economics*, 23(6):695–729.

Radner, R. (1992). Hierarchy: The economics of managing. *Journal of Economic Literature*, 30(3):1382–1415.

Rossi-Hansberg, E., Sarte, P.-D., and Owens III., R. (2009). Firm fragmentation and urban patterns. *International Economic Review*, 50(1):143–186.

Spanos, G. (2019). Firm organization and productivity across locations. *Journal of Urban Economics.*

Strauss-Kahn, V. and Vives, X. (2009). Why and where do headquarters move? *Regional Science and Urban Economics*, 39(2):168–186.

Tate, G. and Yang, L. (2015). The bright side of corporate diversification: Evidence from internal labor markets. *The review of financial studies*, 28(8):2203–2249.

Tian, L. (2017). Division of Labor and Extent of the Market: Theory and Evidence from Brazil. Work in Progress.

Tintelnot, F. (2017). Global production with export platforms. *The Quarterly Journal of Economics*, 132(1):157–209.

Spatial Wage Differentials, Geographic Frictions and the Organization of Labor within Firms

Camilo Acosta, Universidad EAFIT, Colombia; Inter-American Development Bank.
Ditte Håkonsson Lyngemark, Office of Housing and Construction, the Danish Social and Housing Authority, Denmark.

Appendix
(for online publication)

A Extra Tables

Table A1: Descriptive Statistics

Variable	SE Firms Mean	SE Firms Median		ME Firms Mean	ME Firms Median
Establishments	1	1		4.09	2
Avg Distance to HQ (Km)	3.58	2.92		84.63	76.42
Distance to HQ (Km)			non-HQ	140.05	96.92
Managers/Workers	0.10	0.08		0.11	0.09
			HQ	0.11	0.08
			non-HQ	0.11	0.09
Managers	2.64	1		11.48	5
			HQ	6.32	3
			non-HQ	1.84	1
Managers (HQ/esta)			non-HQ	9.74	4
Workers	36.41	20		206.71	64
			HQ	107.23	37
			non-HQ	41.12	13
Workers (HQ/esta)			non-HQ	14.45	4.84
Relative Wage (Raw)	1.67	1.51		1.70	1.60
			HQ	1.88	1.75
			non-HQ	1.51	1.37
Relative Wage (Adjusted)	1.07	0.96	HQ	1.13	1.05
			non-HQ	1.11	1.02
log(Revenue)	10.04	9.87		11.36	11.20
log(Labor Prod)	7	6.94		7.16	7.05
log(Pop density)	5.79	5.04	HQ	5.92	5.18
			non-HQ	5.47	4.98
Sectoral share (municip)	2.60	1.94	HQ	2.77	1.88
			non-HQ	2.30	1.54

Notes: This tables present the mean and median for some of the variables in our dataset. $N_{SE} = 127,247$, $N_{ME,HQ} = 28,111$, $N_{ME,nHQ} = 62,744$. $N = 218,102$. The number of observations is lower for the firm accounting records as they are available only starting in 1999. The first two columns present the statistics for the single-establishment firms. The third and fourth column present the statistics for multi-establishment firms (as a whole) and, where applicable, for HQ and non-HQ establishments separately.

Table A2: Wage Regressions

	Log(Hourly Wage)	
	Manager	Non-Manager
Individual Characteristics	(1)	(2)
Woman	-0.237***	-0.126***
	(0.002)	(0.000)
Region of Origin		
EU + Nordic	0.053***	0.032***
	(0.010)	(0.001)
Less Developed Europe	-0.271***	0.026***
	(0.061)	(0.006)
Other Europe	0.051***	0.037***
	(0.005)	(0.001)
Africa	-0.031*	0.030***
	(0.018)	(0.001)
North America	0.155***	0.129***
	(0.016)	(0.004)
South and Central America	-0.018	0.031***
	(0.021)	(0.002)
Japan	0.183***	0.087***
	(0.062)	(0.012)
Other Asia	-0.142***	0.037***
	(0.010)	(0.001)
Australia and Oceania	0.138***	0.095***
	(0.031)	(0.005)
Unknown	-0.114	0.038***
	(0.094)	(0.005)
Education*(Experience,Tenure)		
Experience	-0.023***	0.008***
	(0.003)	(0.000)
$Experience^2$	0.001***	-0.000***
	(0.000)	(0.000)
Tenure	0.022***	0.024***
	(0.004)	(0.001)
$Tenure^2$	-0.000***	-0.001***
	(0.000)	(0.000)
Primary/Secondary	-0.651***	-0.255***
	(0.019)	(0.003)
$*Experience$	0.051***	0.027***
	(0.003)	(0.000)
$*Experience^2$	-0.001***	-0.001***
	(0.000)	(0.000)
$*Tenure$	0.006*	-0.011***
	(0.004)	(0.001)
$*Tenure^2$	-0.000	0.000***
	(0.000)	(0.000)
Vocational	-0.460***	0.001
	(0.019)	(0.003)
$*Experience$	0.032***	0.004***
	(0.003)	(0.000)
$*Experience^2$	-0.001***	-0.000***
	(0.000)	(0.000)
$*Tenure$	0.002	-0.010***
	(0.004)	(0.001)
$*Tenure^2$	-0.000	0.000***
	(0.000)	(0.000)
Short/Medium Cycle	-0.341***	-0.083***

	(0.018)	(0.003)
$*Experience$	0.037***	0.018***
	(0.003)	(0.000)
$*Experience^2$	-0.001***	-0.000***
	(0.000)	(0.000)
$*Tenure$	0.005	-0.007***
	(0.004)	(0.001)
$*Tenure^2$	-0.000	0.000***
	(0.000)	(0.000)
Long Cycle	-0.323***	-0.014***
	(0.019)	(0.003)
$*Experience$	0.043***	0.024***
	(0.003)	(0.000)
$*Experience^2$	-0.001***	-0.000***
	(0.000)	(0.000)
$*Tenure$	0.008**	-0.005***
	(0.004)	(0.001)
$*Tenure^2$	-0.000**	0.000***
	(0.000)	(0.000)
Constant	6.070***	5.623***
	(0.022)	(0.006)
Disco FE	YES	YES
Pstill FE	YES	YES
Establishment*Year FE	YES	YES
Number of Establishments*Year	298,290	298,290
Observations	825,342	13,222,173
R-squared	0.173	0.329

Notes: This tables present the estimated coefficients for the regressions that estimate the wage indices. Robust standard errors in parentheses. The omitted categories are Denmark (for region of origin) and unknown education. *** p <0.01, ** p <0.05, * p <0.1

Table A3: Relative Labor Demand Estimation
Seemingly Unrelated Regressions (SUR)

	$d\log\left(\frac{M}{L}\right)$ (1)	$\log\left(\frac{L_h}{L_i}\right)$ (2)	$\log\left(\frac{M_h}{M_i}\right)$ (3)
$d\log\left(\frac{v}{w}\right)$	-2.943***		
	(0.720)		
$d\log\left(\frac{w_h}{w_i}\right)$		-3.908***	
		(0.821)	
$d\log\left(\frac{v_h}{v_i}\right)$			-0.2002
			(0.3881)
N		263,621	
Number of Establishments		29,224	

Notes: This table shows the results from estimating a system of regressions of within- and across-establishment relative labor demand as seemingly unrelated regressions (SUR), using relative immigration shocks as instrumental variables. Standard errors clustered by establishment's municipality in parentheses. All regressions include establishment and year fixed effects, and sector*commuting area pre-trends. *** p <0.01, ** p <0.05, * p <0.1

Table A4: Distance to HQ, Copenhagen Dummy and Predicted Immigration

Predicted Imm.	(1) $\log(d_{ih}+1)$	(2) $\log(d_{ih}+1)$	(3) $\log(d_{ih}+1)$	(4) CPH_h	(5) CPH_h	(6) CPH_h
$IS^F_{Mht} - IS^F_{Lht}$	-0.000			-0.000		
	(0.002)			(0.000)		
$IS^F_{Lht} - IS^F_{Lit}$		0.000			0.001**	
		(0.002)			(0.000)	
$IS^F_{Mht} - IS^F_{Mit}$			0.002			0.005**
			(0.008)			(0.002)
N			263,621			
Number of Esta.			29,224			

Notes: Standard errors clustered by establishment's municipality in parentheses; i and h denote the location of the establishment i and its HQ h, respectively. All regressions include year, establishment and sector times HQ commuting area fixed effects. *** p <0.01, ** p <0.05, * p <0.1

Table A5: Parameter Estimates - First Stage of Structural Estimation

Parameter	Workers	Eff. Units
a_1	-0.0007***	-0.0006***
	(0.0002)	(0.0002)
a_2	0.0002***	0.0002***
	(0.0000)	(0.0000)
a_3	0.0013***	0.0013***
	(0.0002)	(0.0002)
a_4	0.0002***	0.0002***
	(0.0000)	(0.0000)
a_5	0.0011***	0.0011***
	(0.0002)	(0.0002)

Notes: Standard errors clustered by establishment's municipality in parentheses; table shows the first stage parameter estimates and standard errors from the two specifications of the structural model from Table 5. *** p <0.01, ** p <0.05, * p <0.1

Table A6: Parameter Estimates

Parameter	Description	Counts
α	Labor Share–Cobb-Douglas	0.395***
		(0.053)
β	Managerial Share–Cobb-Douglas	0.355***
		(0.036)
c_1	Distance Elasticity 1996-97	0.004***
		(0.001)
c_2	Distance Elasticity 1998-99 (Rel. c_1)	-0.003
		(0.002)
c_3	Distance Elasticity 2000-01 (Rel. c_1)	-0.001
		(0.002)
c_4	Distance Elasticity 2002-03 (Rel. c_1)	-0.002
		(0.003)
c_5	Distance Elasticity 2004-05 (Rel. c_1)	0.007**
		(0.003)
c_6	Distance Elasticity 2006-07 (Rel. c_1)	0.0144***
		(0.004)
c_7	Distance Elasticity 2008-09 (Rel. c_1)	0.031***
		(0.005)
c_8	Distance Elasticity 2010-11 (Rel. c_1)	0.037***
		(0.006)
c_9	Distance Elasticity 2012-13 (Rel. c_1)	0.041***
		(0.006)
c_{10}	Distance Elasticity 2014-15 (Rel. c_1)	0.043***
		(0.007)
μ_1	HQ-manager productivity 1996-00	0.002***
		(0.001)
μ_2	HQ-manager productivity 2001-05 (Rel. μ_1)	-0.001
		(0.002)
μ_3	HQ-manager productivity 2006-10 (Rel. μ_1)	-0.017***
		(0.004)
μ_4	HQ-manager productivity 2011-16 (Rel. μ_1)	-0.027***
		(0.005)

Notes: Standard errors clustered by establishment's municipality in parentheses; table shows the parameter estimates and standard errors from two specifications of the structural model. Exact estimation equations can be found in Section 5. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

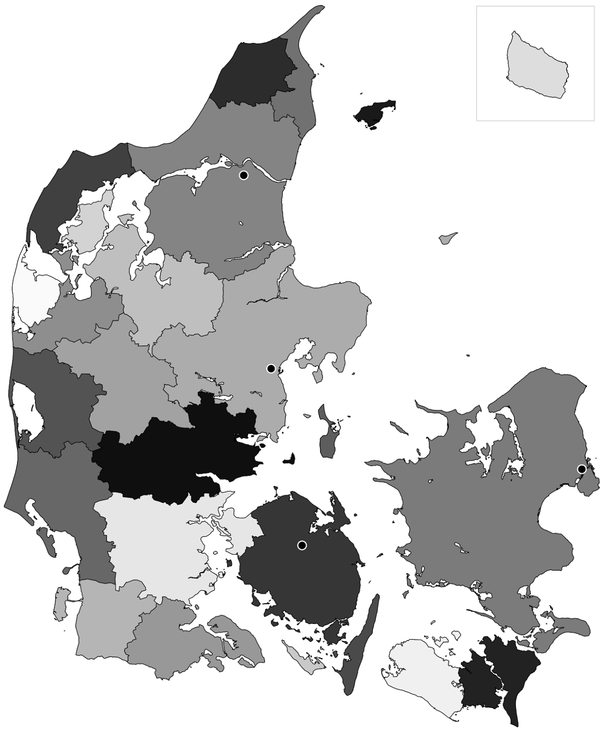
Table A7: Decomposition of the growth of M/L, by periods

Changes in:	1996-2001	2001-2006	2006-2011	2011-2016	1996-2016	Total Change
Within-est. Wage Diff	-7.5%	9%	13.4%	-101.7%	-4.8%	-1.1%
Mgrial Wage Gap	71.9%	60.9%	38.2%	152.1%	66.1%	15.5%
Worker Wage Gap	1.3%	-1.3%	0.5%	-2.9%	0.1%	0%
Distance Elasticity	34.3%	31.3%	47.8%	52.5%	38.6%	9%
Total Change	100%	100%	100%	100%	100%	

Notes: This table shows the results of a decomposition of the growth of the manager-to-workers ratio at the HQ relative to non-HQs, relative to 1996, into the different components suggested by equation (15). The components were calculated using the parameters from Table 5. Each component was regressed on year and year*HQ fixed effects.

B Extra Figures

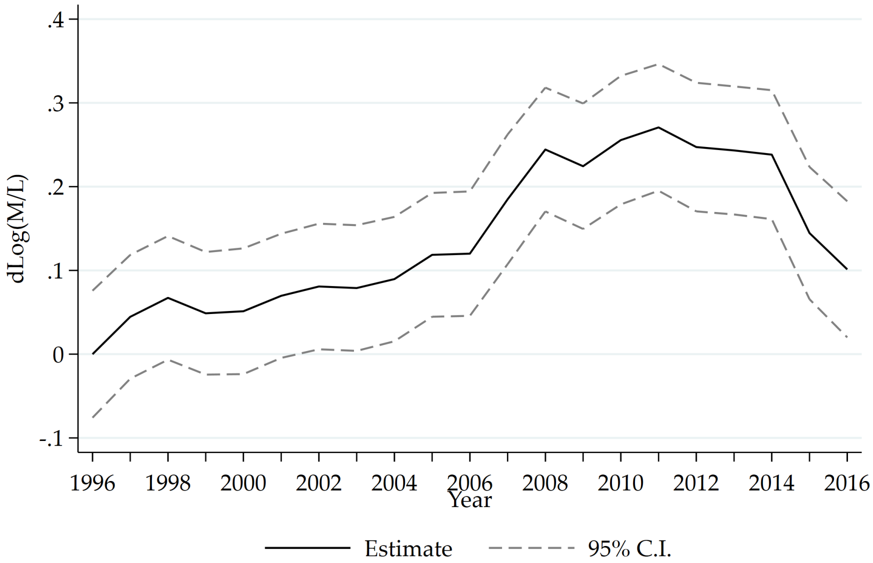
Figure A1: Commuting Areas



This figure shows the commuting areas as defined by Nielsen (2005) based on commuting flows across municipalities in 2004. The black dots represent the four main cities, with Copenhagen being the easternmost point.

Figure A2: Changes in the Ratio of Managers to Workers

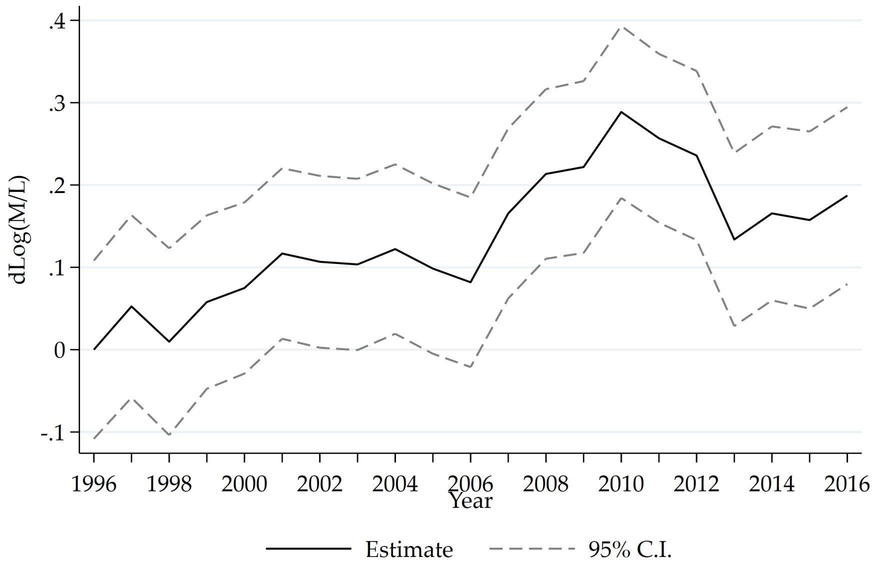
Balanced Panel of Establishments - HQ relative to non-HQ



This figure shows the HQ*year fixed effects from a regression of the log ratio of the establishments' managers to workers on establishment, year and HQ*year fixed effects, using only a balanced panel of establishments.

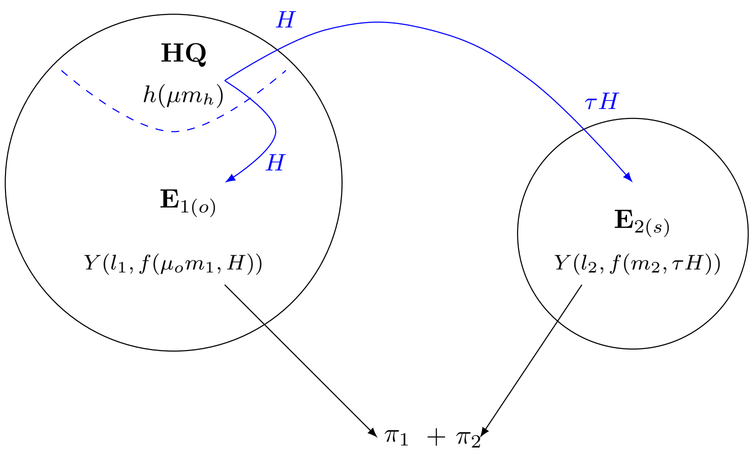
Figure A3: Changes in the Ratio of Managers to Workers

Excluding Multinational Companies - HQ relative to non-HQ



This figure shows the HQ*year fixed effects from a regression of the log ratio of the establishments' managers to workers on establishment, year and HQ*year fixed effects, excluding those companies that in years xxx-xxx reported to have an establishment outside Denmark.

Figure A4: Production Structure in a Two-Establishment Firm



This figure shows a representation of the production structure in two-establishment firm. The left circle represent the firm's HQ and the right circle represents the satellite establishment.

C Geography of Denmark

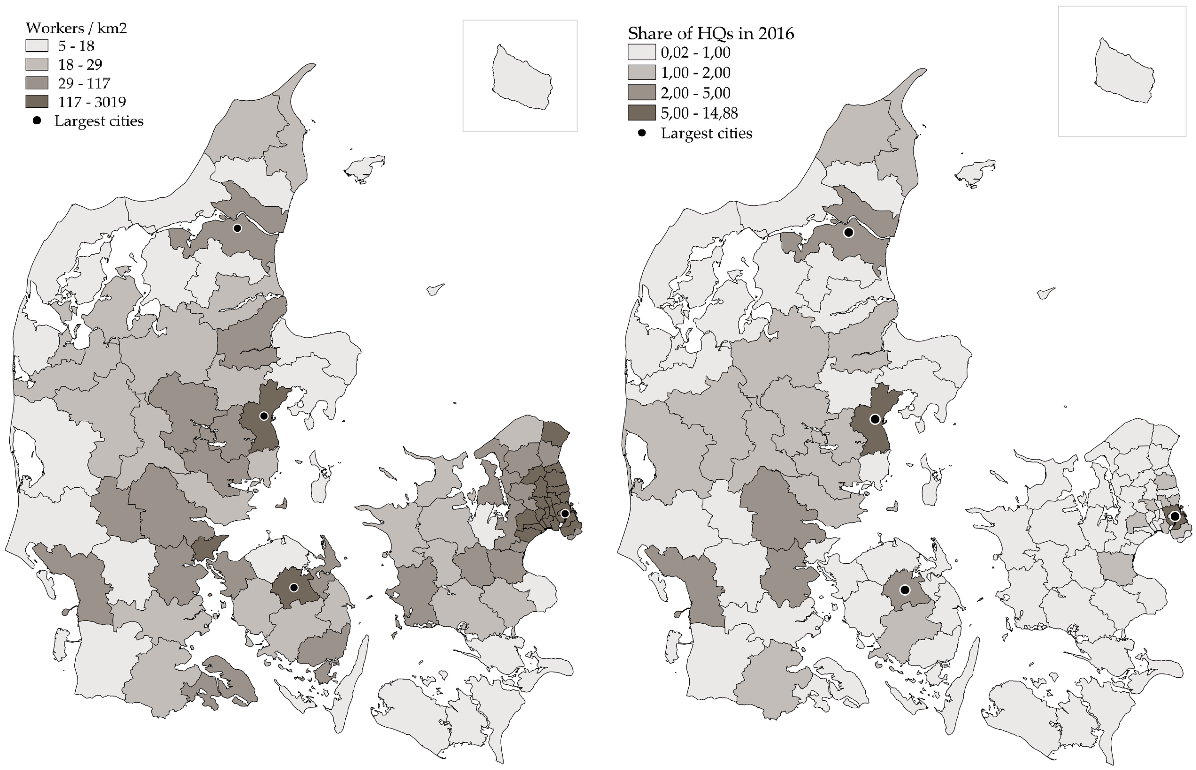
The geographic distribution of population and employment across Danish municipalities is notably uneven. Approximately 36% of the workforce is located in the metropolitan area of Copenhagen, the capital and main city in Denmark. Aarhus, Aalborg and Odense are three medium-size cities that contain 7.1%, 4% and 3.7% of the workforce, respectively. The other half of employment is generated inside the other 94 municipalities, all of which have fewer than 100,000 workers.[22] This distribution can be seen in the left panel of Figure A5, in which we plot the employment density by municipality in 2016. The four black points in this figure represent the four largest cities, with Copenhagen being the easternmost point.

These geographic disparities are also evident when analyzing some firm organization patterns. In particular, in the right panel of Figure A5 we map the distribution of headquarters establishments across municipalities in 2016. This map shows the high concentration of firms' headquarters around Copenhagen, with Aarhus, Aalborg, Odense and the Triangle Region (*Trekantomraadet*) also displaying a relatively high concentration of headquarters. In particular, the commuting areas of the four main municipalities account for 69% of headquarters of multi-establishment firms. This concentration of HQ establishments is not unique to Denmark. Strauss-Kahn and Vives (2009) show that HQ establishments in the United States are disproportionally located in denser and the largest metropolitan areas.

[22]Denmark is divided into 98 municipalities. In 2007, there was a structural reform in the Danish public sector. With this reform, the number of municipalities went from 275 to 98. We account for this by tracking the present municipality boundaries back and using the 98 municipalities for the entire period. Furthermore, we treat Copenhagen and Frederiksberg as one municipality, as the latter is a geographic enclave of the former. This has been done in other articles using similar data, such as Foged and Peri (2016).

Figure A5: Geography of Denmark

Employment Density (2016) — Share of HQs (2016)



The left panel shows the number of workers in 2016 per square kilometer by municipality. The right panel shows the distribution of headquarters across municipalities. Black dots represent the four main cities, with Copenhagen being the easternmost point.

D Mathematical Appendix

D.1 Model Derivations

In this appendix, we present the main derivations from our model. Recall that the model starts from the following production function:

$$Y_{ij} = A_i L_i^{\alpha} \left[\min\{\lambda \mu_j m_i, \theta \tau_j H\}\right]^{\beta},$$

where $\alpha, \beta, \in (0,1)$ and $\alpha + \beta < 1$, and $\mu_o > 1$ and $\mu_s = 1$, and $\lambda > 0$ and $\theta > 0$ are technology-determined constants.

Centralized Solution

Define the total number of managers in the establishment as $M_1 = m_1 + m_h$. With a single-establishment firm, and given that the HQ's location is fixed at location o, the demand for both types of workers and the firm's total profits is given by:

$$\begin{aligned}
M_1^I &= \left[\alpha^{\alpha}\beta^{1-\alpha}\left(\frac{\theta\lambda}{\theta+\lambda}\right)^{\beta}\mu^{\beta}w_o^{-\alpha}v_o^{\alpha-1}pA_i\right]^{\frac{1}{\kappa}}, \\
L_1^I &= \left[\alpha^{1-\beta}\beta^{\beta}\left(\frac{\theta\lambda}{\theta+\lambda}\right)^{\beta}\mu^{\beta}w_o^{\beta-1}v_o^{-\beta}pA_i\right]^{\frac{1}{\kappa}}, \\
\Pi_o^I &= \kappa\left[\alpha^{\alpha}\beta^{\beta}\left(\frac{\theta\lambda}{\theta+\lambda}\right)^{\beta}w_o^{-\alpha}\left(\frac{v_o}{\mu}\right)^{-\beta}pA_i\right]^{\frac{1}{\kappa}} - r_o, \qquad (23)
\end{aligned}$$

where the superscript I indicates that the equations correspond to the solution of a single establishment firm, and $\kappa = 1 - \alpha - \beta$. An interior solution exists as long as $\alpha + \beta \in (0,1)$. This solution shows us that single-establishment firms hire more of both types of workers when there are higher agglomeration economies, lower input prices, or a higher price for the final good, which can be thought of as a proxy for the demand faced by the firm. Moreover, more productive firms hire more workers and are more profitable, as expected.

Fragmented Solution

Define the total number of managers in each establishment as $M_1 = m_1 + m_h$ and $M_2 = m_2$, respectively. When the firm has two establishments, with HQ in o and satellite establishment in $j \in \{o, s\}$, the solution of the firm's problem in equation (6) results in the following demand for both types of workers at each establishment and the firm's total profits:

$$M_{1o}^{II} = \left(\alpha^{\alpha}\beta^{1-\alpha}pA_i\right)^{\frac{1}{\kappa}}\left\{\left(\lambda^{\beta}\mu^{\beta}w_o^{-\alpha}v_o^{\alpha-1}\right)^{\frac{1}{\kappa}} + \frac{1}{\theta\tau_j\mu}\left[\frac{w_j^{-\alpha}}{\left(\frac{v_j}{\lambda\mu_j}+\frac{v_o}{\theta\tau_j\mu}\right)^{1-\alpha}}\right]^{\frac{1}{\kappa}}\right\}, \qquad (24)$$

$$L_{1o}^{II} = \left(\alpha^{1-\beta}\beta^{\beta}w_o^{\beta-1}\left(\frac{v_o}{\lambda\mu}\right)^{-\beta}pA_i\right)^{\frac{1}{\kappa}}, \qquad (25)$$

$$M_{2j}^{II} = \frac{1}{\lambda\mu_j}\left[\alpha^{\alpha}\beta^{1-\alpha}w_j^{-\alpha}\left(\frac{v_j}{\lambda\mu_j}+\frac{v_o}{\theta\tau_j\mu}\right)^{\alpha-1}pA_i\right]^{\frac{1}{\kappa}}, \tag{26}$$

$$L_{2j}^{II} = \left(\alpha^{1-\beta}\beta^{\beta}w_j^{\beta-1}\left(\frac{v_j}{\lambda\mu_j}+\frac{v_o}{\theta\tau_j\mu}\right)^{-\beta}pA_i\right)^{\frac{1}{\kappa}}, \tag{27}$$

$$\Pi_{(o,j)}^{II} = (\tilde{\kappa}pA_i)^{\frac{1}{\kappa}}\left\{\left[w_o^{-\alpha}\left(\frac{v_o}{\lambda\mu}\right)^{-\beta}\right]^{\frac{1}{\kappa}}+\left[w_j^{-\alpha}\left(\frac{v_j\tau_j}{\lambda\mu_j}+\frac{v_o}{\theta\mu}\right)^{-\beta}\tau_j^{\beta}\right]^{\frac{1}{\kappa}}\right\}-r_o-r_j, \tag{28}$$

where the superscript II indicates that the equations correspond to the solution of a two-establishment firm; $\Pi_{o,j}^{II}$ denotes the profits of a firm with two establishments located in o and $j \in \{o,s\}$, respectively; $\kappa = 1-\alpha-\beta$ and $\tilde{\kappa} = \kappa\alpha^{\alpha/\kappa}\beta^{\beta/\kappa}$. Several things can be seen here. First, a reduction in communication costs ($\uparrow \tau$) leads to a increase in the demand for both types of workers in the satellite establishment, as well as an increase in the firm's profits. Second, higher wages lead to a decrease in factor demands and in the profitability of the firm. Land prices also lead to a decrease in the firm's profit. Third, demand for both types of workers in the satellite establishment depends not only on the price of workers and managers in that establishments, but also on the price of HQ managers. This is because production of the final good at any establishment requires HQ services, which is produced by headquarters-type managers. Moreover, notice that an increase in the price of HQ managers leads to a decrease in the demand for both managers and workers at both establishments. Fourth, stronger decreasing returns to scale lead to lower profits and lower factor demand.[23]

Number of Establishments and their Locations

In the first stage of the firm's problem, it has to choose the number of establishments and their respective locations. Given the model's current setting, this decision is equivalent to choosing between having only one establishment, opening a second in o, or opening a second in s. The solution to this problem comes from comparing equations (23) and (28). For instance, a firm will choose to have two establishments, with the HQ located in o and the second establishment located in $j \in \{o,s\}$, if $\Pi_{(o,j)}^{II} > \Pi_o^{I}$ and $\Pi_{(o,j)}^{II} > \Pi_{(o,k)}^{II}$, for $k \neq j, k \in \{o,s\}$.

In particular, a firm will open its satellite establishment in s if the marginal benefit of having that establishment is larger than the extra fixed cost of opening it. This marginal benefit depends on a trade-off between wage differences, communication costs and agglomeration advantages. On the one hand, wage differences between a firm's HQ and other locations create an incentive for the firm to open a new establishment in places where wages are relatively low. On the other hand, by opening new establishments firms incur extra communications cost and could miss out on the productivity advantages present at the HQ location. Deeper comparative statics are explored below.

D.2 Model Predictions and Main Theoretical Results

In this subsection, we present the main results of our model. In particular, we show how the firm's relative labor demands across and within establishments change when wages and

[23] The model can be generalized to include more establishments. In particular, the managerial price aggregator $\left(\frac{v_j\tau_j}{\lambda\mu_j}+\frac{v_o}{\theta\mu}\right)$ would now include a term to account for the price of managers in the other establishments.

communication costs change. For ease of presentation, let us assume that the firm locates its second establishment in $j = s$.

Labor composition within Establishments

Start by analyzing the solution of the labor demand problem of a single-establishment firm from equation (23). The ratio of manager to workers for this firm is given by

$$\left(\frac{M_1}{L_1}\right)_{SE} = \frac{\beta}{\alpha}\frac{w_o}{v_o}. \tag{29}$$

Notice that this ratio depends only on the establishment's relative wage between managers and workers. Therefore, when a single-establishment firm faces an exogenous shock that makes managers more expensive relative to workers, it can only respond by hiring fewer managers and/or more workers. This is not the case for establishments that belong to a multi-establishment firm. First, consider the ratio of managers to workers at non-HQ establishments given by equations (26) and (27):

$$\left(\frac{M_2}{L_2}\right)_{ME} = \frac{\beta}{\alpha}\frac{w_s}{v_s}\left(1 + \frac{v_o}{v_s}\frac{\lambda}{\theta\tau\mu}\right)^{-1}. \tag{30}$$

Just as single-establishment firms, this ratio depends negatively on the relative wages between managers and workers within the establishment. However, notice that it also depends negatively on the price of managers at the HQ relative to the establishment. This implies that an exogenous increase in the price of HQ managers raises the cost of the managerial inputs used at the non-HQ establishment, decreasing the demand for local managers. Finally, note that lower communication costs (τ) and higher HQ agglomeration economies magnify this effect.

Now, consider the ratio of manager to workers at the HQ given by equations (24) and (25):

$$\left(\frac{M_1}{L_1}\right)_{ME} = \frac{\beta}{\alpha}\frac{w_o}{v_o}\left\{1 + \left[\left(\frac{\theta\tau}{\lambda}\right)^{\beta}\left(\frac{w_o}{w_s}\right)^{\alpha}\left(\frac{v_s}{v_o}\frac{\theta\tau\mu}{\lambda} + 1\right)^{\alpha-1}\right]^{\frac{1}{1-\alpha-\beta}}\right\}. \tag{31}$$

In addition to the standard labor demand channel, this ratio also depends on the wage gap of workers and managers between both locations, communication costs and agglomeration economies. This equation suggests that lower wages at a non-HQ establishment would lead to a higher manager-to-workers ratio at at the HQ. Moreover, lower communication costs magnify the effect that differences in the wages of managers and workers have on the HQ's managerial intensity. Finally, lower communication costs themselves have an ambiguous effect on managerial intensity at the HQ. On one hand, since non-HQ establishments receive more H, there will be an incentive to produce more HQ services and hire more managers at the HQ. On the other hand, if the establishment receives a higher quantity of HQ services, it also needs to hire more local managers, which would drive down total profits and the demand for H.

It is evident now that when facing an exogenous wage shock, multi-establishment firms have an extra margin of adjustment that single-establishment firms do not have: They can substitute labor across locations. To see this more clearly, consider first an exogenous shock that renders workers (non-managers) at the central location more expensive. Given this shock, a multi-establishment firm has two options: It can either substitute workers for managers within the establishment or it can substitute workers from the HQ to the non-HQ establishment. Either of these responses would cause a decrease in the number of workers at the HQ (L_o), leading to an increase in HQ managerial intensity.

Consider now an exogenous shock that renders managers in the central location more expensive. Given that HQ managers are used in the production of both the final good and HQ services (which are used by all establishments), the direction of this effect is not as straightforward as the previous one. Using a Slutsky-type decomposition, the total effect of an exogenous increase in the wage of managers at the HQ location on the manager-to-worker ratio at the HQ can be decomposed as the sum of four effects:

$$\frac{\partial M_1/L_1}{\partial v_o} = \frac{\partial (M_1/L_1)^c}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial v_o}$$

$$\begin{aligned} \frac{\partial M_1/L_1}{\partial v_o} &= \underbrace{\frac{\partial (M_1/L_1)^c}{\partial (v_o/w_o)} \cdot \frac{\partial (v_o/w_o)}{\partial v_o}}_{\text{Standard Subst. Effect}} + \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/w_o)} \cdot \frac{\partial (v_o/w_o)}{\partial v_o}}_{\text{Standard Scale Effect}} \\ &+ \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1} \cdot \frac{\partial Y_1}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2} \cdot \frac{\partial Y_2}{\partial (v_o/v_s)} \cdot \frac{\partial (v_o/v_s)}{\partial v_o}}_{\text{HQ Services / Firm Scale Effect}}, \end{aligned} \quad (32)$$

where the superscript c denotes the conditional labor demand functions that result from the firm's cost minimization problem, and Y_1 and Y_2 correspond to the total amount of output produced at HQ and non-HQ establishments, respectively. In Appendix D.3, we show the details of this decomposition.

The first two terms on the right-hand side are the standard substitution and scale effects from a regular Slutsky-type equation. On one hand, if managers are more expensive, the establishment can substitute managers for workers (substitution effect). On the other hand, since the production of the final output requires both managers and workers, more expensive managers lead to a reduction in the establishment's output, which drives down the number of both types of workers at the HQ (scale effect). Both of these effects lead to a decrease in the ratio of managers to workers.

However, keeping constant the wage differences between managers and workers within the establishment, other effect arises. As HQ managers become more expensive, the firm will relocate both managers and workers to the satellite establishment. As this establishment becomes larger, the demand for HQ-services will also increase, hence increasing the demand for managers at the HQ and the ratio of managers to workers.

These two forces change the ratio of manager to workers at the HQ in different directions. However, it can be shown that the latter effect dominates the former. Therefore, an exogenous shock that renders the price of managers more expensive at the central location can lead to an increase in the HQ's managerial intensity, keeping fixed within-establishment wage differences. Note that, taking together the effects of an increase in the wage ratio v_o/v_s on the manager-to-workers ratio at HQ (equation 31) and non-HQ establishments (equation 30), we can conclude that a wider managerial wage gap leads to a more manager-intensive HQ relative to non-HQ establishments.

We formalize all these results in the next proposition. Formal proof of all propositions are included in a Mathematical Appendix.

Proposition 1. ***Managerial Intensity***

(a) Higher relative wages at the establishment's location ($\uparrow \frac{v}{w}$) lead to less manager-intensive establishments ($\downarrow \frac{M}{L}$). This holds for every establishment in the economy.

(b) For multi-establishment firms, a wider worker wage gap across locations ($\uparrow \frac{w_o}{w_s}$) leads to a more manager-intensive HQ relative to the non-HQ establishment ($\uparrow \frac{M_h/L_h}{M_i/L_i}$).

(c) For multi-establishment firms, a wider managerial wage gap across locations ($\uparrow \frac{v_o}{v_s}$) leads to a more manager-intensive HQ relative to the non-HQ establishment ($\uparrow \frac{M_h/L_h}{M_i/L_i}$). Lower communication costs ($\uparrow \tau$) magnify the effect.

Proof. (a) First, for single-establishment firms the proof is straightforward as:

$$\frac{\partial(M_1/L_1)}{\partial(v_o/w_o)} = -\frac{\beta}{\alpha}\left(\frac{w_o}{v_o}\right)^2 < 0$$

For non-HQ establishments belonging to multi-establishment firms, from equation (30) take the partial derivative of the manager to worker ratio with respect to the respective wage ratio (v_s/w_s):

$$\frac{\partial(M_2/L_2)}{\partial(v_s/w_s)} = -\frac{\beta}{\alpha}\left(\frac{w_s}{v_s}\right)^2\left(1+\frac{v_o}{v_s}\frac{\lambda}{\theta\tau\mu}\right)^{-1} < 0$$

For HQ establishments of multi-establishment firms, from equation (31) take the partial derivative of the manager to worker ratio with respect to the respective wage ratio (v_o/w_o):

$$\frac{\partial(M_1/L_1)}{\partial(v_o/w_o)} = -\frac{\beta}{\alpha}\left(\frac{w_o}{v_o}\right)^2\left\{1+\left[\left(\frac{\theta\tau}{\lambda}\right)^{\beta}\left(\frac{w_o}{w_s}\right)^{\alpha}\left(\frac{v_s}{v_o}\frac{\theta\tau\mu}{\lambda}+1\right)^{\alpha-1}\right]^{\frac{1}{1-\alpha-\beta}}\right\} < 0$$

(b) Start by finding the ratio between equations (31) and (30):

$$\frac{(M_1/L_1)}{(M_2/L_2)} = \frac{(v_s/w_s)}{(v_o/w_o)}\left\{1+\left[\left(\frac{\theta\tau}{\lambda}\right)^{\beta}\left(\frac{w_o}{w_s}\right)^{\alpha}\left(\frac{v_s}{v_o}\frac{\theta\tau\mu}{\lambda}+1\right)^{\alpha-1}\right]^{\frac{1}{1-\alpha-\beta}}\right\}\left(1+\frac{v_o}{v_s}\frac{\lambda}{\theta\tau\mu}\right)$$

The partial derivative of this ratio with respect to w_o/w_s is proportional to:

$$\frac{\partial\frac{(M_1/L_1)}{(M_2/L_2)}}{\partial w_o/w_s} \propto \frac{1-\beta}{1-\alpha-\beta}\left[\left(\frac{\theta\tau}{\lambda}\right)^{\beta}\left(\frac{w_o}{w_s}\right)^{\alpha}\left(\frac{v_s}{v_o}\frac{\theta\tau\mu}{\lambda}+1\right)^{\alpha-1}\right]^{\frac{1}{1-\alpha-\beta}} > 0$$

(c) On one hand, from equation (31) it can be observed that:

$$\frac{\partial(M_1/L_1)}{\partial(v_o/v_s)} > 0$$

On the other hand, from equation (30) it can be observed that:

$$\frac{\partial(M_2/L_2)}{\partial(v_o/v_s)} < 0$$

Therefore, it must be that:

$$\frac{\partial\frac{(M_1/L_1)}{(M_2/L_2)}}{\partial v_o/v_s} > 0$$

□

We can derive the elasticities of the HQ manager-to-worker ratio with respect to a change in the wage gap of either workers or managers across locations:

Corollary 1. *Keeping the within-establishment wage ratio ($\frac{v}{w}$) constant and defining $M_1 = m_1 + m_h$, the elasticities of the HQ ratio of managers to workers, with respect to changes in the wage gap of workers and managers across locations are given by:*

(a) For workers:

$$\frac{\partial \log(M_1/L_1)}{\partial \log(w_o/w_s)} = \frac{\alpha}{1-\alpha-\beta} \cdot \frac{v_o m_h}{v_o M_1} > 0.$$

(b) For managers:

$$\frac{\partial \log(M_1/L_1)}{\partial \log(v_o/v_s)} = \frac{1-\alpha}{1-\alpha-\beta} \cdot \frac{v_s/\lambda}{(v_s/\lambda)+(v_o/\theta\tau\mu)} \cdot \frac{v_o m_h}{v_o M_1} > 0.$$

Labor composition across Establishments

From our model, we can also write two equations that can help us understand the substitution patterns of workers and managers across establishments. Empirically, these two equations are useful for two reasons. First, since these equations will yield relatively simple linear regressions, their reduced-form estimates will help us test some of the model's predictions and, therefore, its validity. Second, the across-location elasticities of substitution implied by these equations provide important variation to identify the structural parameters of our model.

Under our current assumptions, we can write the ratio of workers at the HQ relative to the non-HQ establishment as

$$\frac{L_1}{L_2} = \left[\left(\frac{w_o}{w_s}\right)^{\beta-1}\left(\frac{v_s}{v_o}\mu + \frac{\lambda}{\theta\tau}\right)^{\beta}\right]^{\frac{1}{1-\alpha-\beta}}. \tag{33}$$

Similarly, for managers:

$$\frac{M_1}{M_2} = \frac{\lambda}{\theta\tau\mu} + \frac{1}{\mu}\left[\left(\frac{w_o}{w_s}\right)^{-\alpha}\left(\frac{v_s}{v_o}\mu + \frac{\lambda}{\theta\tau}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha-\beta}}. \tag{34}$$

From these expressions, we highlight three effects. First, lower communication costs lead to a decrease in the ratios of workers and managers at the HQ relative to the other establishment. This is caused by the fact that with lower communication costs, non-HQ establishments receive more services from the HQ and hire more managers and workers to produce more output. Second, higher HQ manager-specific productivity (μ) has a positive effect on the ratio of workers at the HQ relative to the non-HQ, but an ambiguous effect on the managerial ratio. On one hand, since managers are now more productive at the HQ, there is an incentive to increase the number of managers at the HQ relative to the other establishment. However, this rise in productivity also leads to a higher production of HQ services, thus driving up the demand for local managers at the non-HQ establishment.

Third, the share of workers and managers at the HQ depends negatively on the respective wage gap across locations. This result suggests that the size of the HQ relative to the non-HQ establishment, decreases when the wage gap across locations is wider. Even though this is an expected result, it is worth discussing the across-location elasticities of substitution, since this variation will help us identify the model parameters in the estimation from Section 6.2. From the two equations above, we calculate the respective across-location elasticities of substitution of workers and managers as

$$\begin{aligned} \frac{\partial \log(L_1/L_2)}{\partial \log(w_o/w_s)} &= \frac{\beta - 1}{1-\alpha-\beta} < 0, \\ \frac{\partial \log(M_1/M_2)}{\partial \log(v_o/v_s)} &= \frac{\alpha - 1}{1-\alpha-\beta} \cdot \frac{v_s/\lambda}{(v_s/\lambda) + (v_o/\theta\mu\tau)} \cdot \frac{v_o m_1}{v_o M_1} < 0, \end{aligned} \tag{35}$$

where $M_1 = m_1 + m_h$. While the elasticity of substitution of workers across establishments is constant, this is not the case for managers. The latter elasticity depends on two cost shares: (i) the cost share of non-HQ managers on the total managerial cost of the non-HQ establishment, and (ii) the cost share of production managers on the total managerial costs at the HQ. In particular, if production is not dependent on HQ services, the elasticity converges to a constant given by $\alpha - 1/1 - \alpha - \beta$. Moreover, as the use of HQ services increases, this elasticity converges to zero. Therefore, this elasticity implies that when firms are very dependent on the within-firm public good, we would observe a low degree of substitution of managers across establishments for a given exogenous change in the wage gap of managers. Furthermore, if $\beta < \alpha$, the across-location elasticity of substitution for workers will always be more negative than the one for managers. Finally, consider the interactions between the previous elasticities and communication costs. From equation (35), we observe that lower communication costs make this elasticity more negative, thus magnifying an increase in the managerial wage gap.[24] These results can be summarized as:

Proposition 2. ***Substitution Across Locations***

(a) A steeper wage gap for workers ($\uparrow \frac{w_o}{w_s}$) leads to a lower share of workers at the HQ ($\downarrow \frac{L_h}{L_i}$).

(b) A steeper wage gap for managers ($\uparrow \frac{v_o}{v_s}$) leads to a lower share of managers at the HQ ($\downarrow \frac{M_h}{M_i}$). Lower communication costs ($\uparrow \tau$) magnify the effects.

Proof. (a) Consider equation (33). Take logs and its partial derivative with respect to $\log(w_o/w_s)$:

$$\frac{\partial \log(L_1/L_2)}{\partial \log(w_o/w_s)} = (\beta - 1) < 0$$

(b) Consider equation (34). That equation is proportional to:

$$\frac{M_1}{M_2} \propto \frac{1}{\mu} \left[\left(\frac{w_o}{w_s} \right)^{-\alpha} \left(\frac{v_s}{v_o} \mu + \frac{\lambda}{\theta\tau} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha-\beta}}$$

Taking logs of both sides of the equations and its partial derivative with respect to $\log(v_o/v_s)$:

$$\frac{\partial \log(M_1/M_2)}{\partial \log(v_o/v_s)} \propto -\frac{1-\alpha}{1-\alpha-\beta} \cdot \frac{1}{\frac{v_o}{v_s} + \frac{\lambda}{\theta\tau\mu} \left(\frac{v_o}{v_s} \right)^2} < 0$$

Moreover, notice that a larger τ makes the previous expression even more negative. □

[24] This is a result of the assumed complementarity between local managers and HQ services. In Appendix D.4, we show that the direction of this cross-derivative is zero if the managerial bundle is we assume a Cobb-Douglas production function for the managerial bundle.

Number of establishments

All of the comparative statics so far have assumed that the firm has two establishments, located in o and s. Nevertheless, we would like to know how the firm's decision on whether to have a second establishment, and its eventual location, changes with wages, communication costs and productivity. Recall that these decisions involve fixed costs, which in our model are connected to the price of local land. Using equations (23) and (28), our model gives the following prediction regarding the change in the firm's number of establishments:

Proposition 3. *The average number of establishments per firm increases with either lower communication costs, higher wage gaps (for either type of labor), higher agglomeration economies in the central location, higher firm-specific productivity, or higher differences in the price of land.*

Proof. Taking the difference between equations (28) and (23) yields the following expression:

$$\begin{aligned}\Delta\Pi &= \Pi^{II}_{(o,s)} - \Pi^{I}_{o} \\ &= (\tilde{\kappa}pA)^{\frac{1}{\kappa}}\left\{\left[w_o^{-\alpha}\left(\frac{v_o}{\lambda\mu}\right)^{-\beta}\right]^{\frac{1}{\kappa}}\left[1-\left(\frac{\theta}{\theta+\lambda}\right)^{\frac{\beta}{\kappa}}\right]+\left[w_s^{-\alpha}\left(\frac{v_s\tau}{\lambda}+\frac{v_o}{\theta\mu}\right)^{-\beta}\tau^{\beta}\right]^{\frac{1}{\kappa}}\right\} - r_s\end{aligned}$$

Rewrite this expression as:

$$\Delta\Pi = \Phi_1 + \Phi_2 - r_s,$$

where Φ_1 and Φ_2 are given by their respective expressions in the equation above. Proceeding with the comparative statics:

(a) Communication costs (τ): notice that:

$$\frac{\partial\Delta\Pi}{\partial\tau} = \frac{\partial\Phi_2}{\partial\tau} = \Phi_2\frac{\beta}{\kappa\tau}\frac{\frac{v_o}{\theta\mu}}{\frac{v_s\tau}{\lambda}+\frac{v_o}{\theta\mu}} > 0$$

(b) Agglomeration economies in the central location (μ):

$$\frac{\partial\Delta\Pi}{\partial\mu} = \frac{\partial\Phi_1}{\partial\mu} + \frac{\partial\Phi_2}{\partial\mu} = \Phi_1\frac{\beta}{\kappa\mu} + \Phi_2\frac{\beta}{\kappa\mu}\frac{\frac{v_o}{\theta\mu}}{\frac{v_s\tau}{\lambda}+\frac{v_o}{\theta\mu}} > 0$$

(c) Higher firm-specific productivity (A) and higher differences in the price of land (r_s): for both of these variables it is clear that: $\frac{\partial\Delta\Pi}{\partial A} > 0$ and $\frac{\partial\Delta\Pi}{\partial r_s} < 0$.

(d) For both the wage gaps for workers (w_o/w_s) and the wage gaps for managers (v_o/v_s), a simple analytic expression cannot be found. Nonetheless, the numeric simulations from Section 6.4 show that higher wage gaps for either type of workers leads to an increase in the average number of establishments.

□

D.3 Slutsky Decomposition of the HQ Managerial Intensity

Consider the cost minimization problem of the two-establishment firm:

$$
\begin{aligned}
\min_{L_1,L_2,m_1,m_2,m_h} \quad w_o L_1 + w_s L_2 \quad &+ \quad v_o m_1 + v_s m_2 + v_o m_h \\
\text{s.t. } \bar{Y}_1 \quad &= \quad L_1^{\alpha}[\min\{m_1, m_h\}]^{\beta} \\
\bar{Y}_2 \quad &= \quad L_2^{\alpha}[\min\{m_2, \tau m_h\}]^{\beta},
\end{aligned}
$$

where we assumed $\lambda = \theta = 1$ and $\mu_o = \mu_s = 1$ without loss of generality. The solution of this problem yields the conditional labor demand for workers and managers at both locations:

$$
\begin{aligned}
L_1^c &= \left[\bar{Y}_1 \left(\frac{\beta}{\alpha}\frac{w_o}{v_o}\right)^{-\beta}\right]^{\frac{1}{\alpha+\beta}}, &(36)\\
L_2^c &= \left[\frac{Y_2}{\tau^{\beta}} \left(\frac{\beta}{\alpha}\frac{w_s}{v_o}\right)^{-\beta}\right]^{\frac{1}{\alpha+\beta}}, \\
M_1^c &= \left(\frac{\beta}{\alpha v_o}\right)^{\frac{\alpha}{\alpha+\beta}} \left[\left(\bar{Y}_1 w_o^{\alpha}\right)^{\frac{1}{\alpha+\beta}} + \left(\bar{Y}_2 \tau^{-\beta} w_s^{\alpha}\right)^{\frac{1}{\alpha+\beta}}\right], &(37)\\
M_2^c &= \left[\frac{Y_2}{\tau^{-\alpha}} \left(\frac{\beta}{\alpha}\frac{w_s}{v_o}\right)^{\alpha}\right]^{\frac{1}{\alpha+\beta}},
\end{aligned}
$$

where $M_1^c = m_1^c = m_h^c$ and $M_2^c = m_2^c$. From equations (37) and (36), we can write the conditional relative demand at the HQ as:

$$
\left(\frac{M_1}{L_1}\right)^c = \frac{\beta}{\alpha}\frac{w_o}{v_o} + \frac{\beta}{\alpha}\frac{w_o}{v_o}\left(\frac{\bar{Y}_2}{\bar{Y}_1}\tau^{-\beta}\left(\frac{w_s}{w_o}\right)^{\alpha}\right)^{\frac{1}{\alpha+\beta}}. \qquad (38)
$$

From the profit maximization problem in the text, we can find and write the optimal level of output in both establishments as:

$$
\begin{aligned}
Y_1 &= \left(\alpha^{\alpha}\beta^{\beta}p^{\alpha+\beta}\left(\frac{v_o}{w_o}\right)^{-\alpha}\left(\frac{v_o}{v_s}\right)^{-\alpha-\beta} v_s^{-\alpha-\beta}\right)^{\frac{1}{1-\alpha-\beta}}, &(39)\\
Y_2 &= \left(\alpha^{\alpha}\beta^{\beta}p^{\alpha+\beta}w_s^{-\alpha}\left(1+\frac{v_o}{v_s\tau}\right)^{-\beta} v_s^{-\beta}v_o^{-\beta}\right)^{\frac{1}{1-\alpha-\beta}}. &(40)
\end{aligned}
$$

With all these elements, consider the Slutsky decomposition –equation (32)– from the text:

$$
\begin{aligned}
\frac{\partial M_1/L_1}{\partial v_o} &= \frac{\partial (M_1/L_1)^c}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_1}\cdot\frac{\partial Y_1}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2}\cdot\frac{\partial Y_2}{\partial v_o} \\
&= \underbrace{\frac{\partial (M_1/L_1)^c}{\partial (v_o/w_o)}\cdot\frac{\partial (v_o/w_o)}{\partial v_o}}_{A\equiv\text{Standard Subst. Effect}} + \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1}\cdot\frac{\partial Y_1}{\partial (v_o/w_o)}\cdot\frac{\partial (v_o/w_o)}{\partial v_o}}_{B\equiv\text{Standard Scale Effect}} \\
&+ \underbrace{\frac{\partial (M_1/L_1)^c}{\partial Y_1}\cdot\frac{\partial Y_1}{\partial (v_o/v_s)}\cdot\frac{\partial (v_o/v_s)}{\partial v_o} + \frac{\partial (M_1/L_1)^c}{\partial Y_2}\cdot\frac{\partial Y_2}{\partial (v_o/v_s)}\cdot\frac{\partial (v_o/v_s)}{\partial v_o}}_{C\equiv\text{Firm Scale Effect}}.
\end{aligned}
$$

Terms A, B and C can be written as:

$$
\begin{aligned}
A &= \frac{-\beta}{\alpha}\frac{w_o}{v_o^2}\left[1+\left(\frac{\bar{Y}_2}{\bar{Y}_1}\tau^{-\beta}\left(\frac{w_s}{w_o}\right)^{\alpha}\right)^{\frac{1}{\alpha+\beta}}\right] < 0,\\
B &= \frac{-\beta}{(\alpha+\beta)(1-\alpha-\beta)}\frac{w_o}{v_o v_s}\left(\frac{\bar{Y}_2}{\bar{Y}_1}\tau^{-\beta}\left(\frac{w_s}{w_o}\right)^{\alpha}\right)^{\frac{1}{\alpha+\beta}} < 0,\\
C &= \frac{\beta}{\alpha(\alpha+\beta)(1-\alpha-\beta)}\frac{w_o}{v_o^2}\left(\frac{\bar{Y}_2}{\bar{Y}_1}\tau^{-\beta}\left(\frac{w_s}{w_o}\right)^{\alpha}\right)^{\frac{1}{\alpha+\beta}} [\underbrace{-\beta\frac{v_o/\tau}{v_s+v_o/\tau}}_{\downarrow\text{Scale non-HQ}} + \underbrace{(\alpha+\beta)}_{\uparrow\text{Scale HQ}}] > 0. \quad (41)
\end{aligned}
$$

Notice that term A corresponds to the direct substitution effect between managers and workers given by an increase in the wage of managers. Term B corresponds to the within-establishment scale or output effect. That is, the increase in the wage of managers leads to an increase in the cost of production, causing less of the good to be sold, further affecting the demand for both types of labor. Finally, term C comes from changes in the wage of managers at the HQ, relative to non-HQ locations, keeping the within-establishment wage ratios fixed. This effect encompasses two scale effects that arise due to the fact that all of the firm's establishments need HQ services in order to produce final output. On one hand, that an increase in the price of HQ managers make production at the non-HQ establishment more expensive since it increases the cost of the managerial bundle. This effect will drive down the demand for local and headquarter managers, driving down M_1/L_1. On the other hand, since non-HQ establishments are now cheaper, the firm wants to make these establishments larger. The increase in the size of the satellite establishments, generates an increase in the demand for HQ services, which are an input necessary for production. Since HQ services are a manager intensive good, this would cause and increase in the demand for managers at the headquarters. It can be seen easily that the latter effect dominates the former.

D.4 Model with a Cobb-Douglas Managerial Bundle

In this appendix, we derive our main theoretical results assuming the following Cobb-Douglas managerial bundle:

$$
Y_{ij} = A_i L_i^{\alpha}\left[(\mu_j m_i)^{\gamma}(\tau_j H)^{1-\gamma}\right]^{\beta}, \quad \gamma < 1.
$$

Centralized Solution

Define the total number of managers in the establishment as $M_1 = m_1 + m_h$. With a single-establishment firm, and given that the HQ's location is fixed at location o, the demand for both types of workers and the firm's total profits is given by:

$$
\begin{aligned}
M_1^I &= \left[\phi_m \cdot \mu^{\beta} w_o^{-\alpha} v_o^{\alpha-1} p A_i\right]^{\frac{1}{\kappa}}\left[1+\frac{(1-\gamma)\mu}{\gamma}\right],\\
L_1^I &= \left[\phi_l \cdot w_o^{\beta-1}\left(\frac{v_o}{\mu}\right)^{-\beta} p A_i\right]^{\frac{1}{\kappa}},
\end{aligned}
$$

$$\Pi_o^I = \kappa\left[\phi_p \cdot w_o^{-\alpha}\left(\frac{v_o}{\mu}\right)^{-\beta} pA_i\right]^{\frac{1}{\kappa}} - r_o,$$

where the superscript I indicates that the equations correspond to the solution of a single establishment firm, and $\kappa = 1-\alpha-\beta$, and ϕ_m, ϕ_l, ϕ_p are constants. An interior solution exists as long as $\alpha+\beta \in (0,1)$. Similar to the case with the fixed proportion managerial bundle, this solution shows us that single-establishment firms hire more of both types of workers when there are higher agglomeration economies, lower input prices or higher price for the final good.

Fragmented Solution

Define the total number of managers in each establishment as $M_1 = m_1 + m_h$ and $M_2 = m_2$. When the firm has two establishments, with HQ in o and satellite establishment in $j \in \{o,s\}$, the optimal demand for both types of workers at each establishment and the firm's total profits can be written as:

$$M_{1o}^{II} = \left[\phi_m \cdot \left(\frac{\mu\mathbb{R}}{v_o}\right)^{(1-\gamma)\beta} pA_i\right]^{\frac{1}{\kappa}}\left[\left(\frac{\mu^{\gamma\beta}}{w_o^{\alpha}v_o^{1-\alpha}}\right)^{\frac{1}{1-\alpha-\gamma\beta}} + \frac{(1-\gamma)\mu\mathbb{R}}{\gamma v_o}\right],$$

$$M_{2j}^{II} = \left[\phi_m \cdot \left(\frac{\mu\mathbb{R}}{v_o}\right)^{(1-\gamma)\beta} pA_i\right]^{\frac{1}{\kappa}}\left(\frac{\tau^{(1-\gamma)\beta}}{w_j^{\alpha}v_j^{1-\alpha}}\right)^{\frac{1}{1-\alpha-\gamma\beta}},$$

$$L_{ij}^{II} = \left[\phi_l \cdot \left(\frac{\mu\mathbb{R}}{v_o}\right)^{(1-\gamma)\beta} pA_i\right]^{\frac{1}{\kappa}}\left(\frac{\mu_j^{\gamma\beta}\tau_j^{(1-\gamma)\beta}}{w_j^{1-\gamma\beta}v_j^{\gamma\beta}}\right)^{\frac{1}{1-\alpha-\gamma\beta}}, \quad i \in \{1,2\},$$

$$\Pi_{(o,j)}^{II} = \kappa\left[\phi_p \cdot \left(\frac{v_o}{\mu}\right)^{-(1-\gamma)\beta}\mathbb{R}^{1-\alpha-\gamma\beta} pA_i\right]^{\frac{1}{\kappa}} - r_o - r_j,$$

where ϕ_m, ϕ_l, ϕ_p are constants, the superscript II indicates that the equations correspond to the solution of a two-establishment firm, $\Pi_{o,j}^{II}$ denotes the profits of a firm with two establishments located in o and $j \in \{o,s\}$, respectively, and $\kappa = 1-\alpha-\beta$. Moreover,

$$\mathbb{R} = \sum_{j\in\mathbb{L}}\left[\frac{\left(\tau_j^{1-\gamma}\mu_j^{\gamma}\right)^{\beta}}{w_j^{\alpha}v_j^{\gamma\beta}}\right]^{\frac{1}{1-\alpha-\gamma\beta}},$$

where $\mathbb{L}$ is the set containing the locations of each of the firm's establishments. In the case of our two-establishment two-location model, $\mathbb{L}$ equals either $\{o,o\}$ or $\{o,s\}$. The expression $\mathbb{R}$ can be thought as a measure of the marginal revenue of an additional manager that works in the production of H. Comparing these equations with the respective equations from the text, it is evident that the comparative statics with respect to wages, communication costs and the agglomeration parameter hold.

D.4.1 Model Predictions

Labor composition within the establishments

The ratio of managers-to-workers for single-establishment firm is still given by:

$$\left(\frac{M_1}{L_1}\right)_{SE} = \frac{\gamma\beta}{\alpha}\frac{w_o}{v_o}.$$

Consider now the ratio of managers to workers at non-HQ establishments from multi-establishment firms. In this case, the ratio is the same as for single-establishment firms, and thus, does not depend on the managerial wage gap across locations:

$$\left(\frac{M_2}{L_2}\right)_{ME} = \frac{\gamma\beta}{\alpha}\frac{w_s}{v_s}.$$

The ratio of managers-to-workers at the HQ is given by:

$$\left(\frac{M_1}{L_1}\right)_{ME} = \frac{\gamma\beta}{\alpha}\frac{w_o}{v_o}\left\{1+\left(\frac{1-\gamma}{\gamma}\right)\left[1+\left(\tau^{(1-\gamma)\beta}\mu^{-\gamma\beta}\left(\frac{w_o}{w_s}\right)^{\alpha}\left(\frac{v_o}{v_s}\right)^{\gamma\beta}\right)^{\frac{1}{1-\alpha-\gamma\beta}}\right]\right\}.$$

Just as in the case with the fixed-proportions managerial bundle, this equation suggests that higher wages at the HQ, relative to the non-HQ establishment, would lead to an increase the manager-to-worker ratio at the HQ. Moreover, lower communication costs magnify these effects. We can also derive elasticities of the HQ ratio of managers-to-workers, with respect to changes in the wage gap of workers and managers across locations. These elasticities are:

$$\frac{\partial \log(M_1/L_1)}{\partial \log(w_o/w_s)} = \frac{\beta(1-\gamma)}{1-\alpha-\gamma\beta}\cdot\frac{w_sL_2}{v_oM_1} > 0,$$

$$\frac{\partial \log(M_1/L_1)}{\partial \log(v_o/v_s)} = \frac{\beta^2\gamma(1-\gamma)}{\alpha(1-\alpha-\gamma\beta)}\cdot\frac{w_sL_2}{v_oM_1} > 0.$$

Labor composition across establishments

Under our current assumptions, we can write the ratio of workers at the HQ, relative to the non-HQ establishment, as:

$$\frac{L_1}{L_2} = \left(\tau^{-(1-\gamma)\beta}\mu^{\gamma\beta}\left(\frac{w_o}{w_s}\right)^{\gamma\beta-1}\left(\frac{v_o}{v_s}\right)^{-\gamma\beta}\right)^{\frac{1}{1-\alpha-\gamma\beta}}.$$

Similarly, for managers:

$$\frac{M_1}{M_2} = \frac{w_o/w_s}{v_o/v_s}\left\{1+\left(\frac{1-\gamma}{\gamma}\right)\left[1+\left(\tau^{(1-\gamma)\beta}\mu^{-\gamma\beta}\left(\frac{w_o}{w_s}\right)^{\alpha}\left(\frac{v_o}{v_s}\right)^{\gamma\beta}\right)^{\frac{1}{1-\alpha-\gamma\beta}}\right]\right\}.$$

Note that the share of workers and managers at the HQ still depends negatively on the respective wage gap across locations. This result suggests that the size of the HQ relative to the non-HQ establishment decreases when the wage gap across locations is wider. From the two equations above, we can calculate the respective across-location elasticities of substitution, of workers and managers, as:

$$\frac{\partial \log(L_1/L_2)}{\partial \log(w_o/w_s)} = \frac{\gamma\beta-1}{1-\alpha-\gamma\beta} < 0,$$

$$\frac{\partial \log(M_1/M_2)}{\partial \log(v_o/v_s)} = -1 - \frac{\beta}{1-\alpha-\gamma\beta} \cdot \frac{v_o m_1}{v_o M_1} < 0,$$

where $M_1 = m_1 + m_h$. While the elasticity of substitution of workers across establishments is constant, this is not the case for managers. The latter elasticity depends on the cost share of production managers on the total managerial costs at the HQ. In particular, if production is not dependent on HQ services, the elasticity converges to a constant given by $-1-(\beta/1-\alpha-\gamma\beta)$. Moreover, as the use on HQ services increases, this elasticity decreases and converges to -1. When comparing these two elasticities with the ones derived in the text for the fixed-proportions managerial bundle, an two important difference comes up: none of the above elasticities depend on the communication costs parameter τ. This theoretical result contradicts our result from Section 6.1, which suggest a significant interaction between changes in wages across locations and the distance between establishments. This evidence also suggests that the model with a fixed-proportion managerial bundle can fit the data better than the one using the Cobb-Douglas function.

E Extensive Margin: Number of Establishments

We are also interested in knowing how changes in wages, communication costs and agglomeration economies affect other dimensions of the observed firm fragmentation patterns. Since our model gives us sharp predictions regarding the firm's number of establishments, we focus on this appendix on the extensive margin decisions. In particular, we use numerical simulations to analyze whether our model captures some of the important regularities we find in the data.

We start by analyzing how distance from HQ, wage differentials and population density affect the probability that a firm has an establishment in a given municipality. Table A8 presents probit regressions between a binary variable that equals 1 if a firm with HQ in k has an establishment in municipality j and different combinations of our variables of interest. For these regressions, we use the whole period of our data (1981-2016). Wages are measured following the methodology described in Section 2, but include a municipality-year fixed effect in the regressions instead of an occupation-establishment-year fixed effect. Thus, the measured wage is the average of all firms in a municipality and is the same for both workers and managers. All of the regressions include sector, year and HQ commuting area fixed effects. We have not included firm fixed effects to avoid problems with incidental parameters. Identification of these estimates comes from comparing, in the cross-section, the location choices of firms within the same sector, HQ commuting area, or year. The estimates presented in this table are merely correlations and do not represent causal relations.

Table A8: Location Probability

	Prob(Firm with HQ in k has an est in j)						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Variables \ Period	1981-2016	1981-2016	1981-2016	1981-1989	1990-1998	1999-2007	2008-2016
Dist to HQ	-0.44***	-0.46***	-0.48***	-0.51***	-0.49***	-0.48***	-0.47***
	(0.026)	(0.024)	(0.024)	(0.030)	(0.025)	(0.025)	(0.022)
$log(W_k/W_j)$		1.98***	-12.30***	-2.924	-13.12***	-14.97***	-15.17***
		(0.688)	(3.167)	(3.582)	(3.663)	(3.087)	(4.033)
$log(W_k/W_j)*$Dist to HQ			2.97***	1.40*	3.31***	3.32***	3.48***
			(0.655)	(0.800)	(0.755)	(0.623)	(0.807)
Pop Density at HQ			0.01	0.00	0.02	0.03	0.01
			(0.023)	(0.021)	(0.022)	(0.028)	(0.024)
N		5,946,391		1,355,187	1,442,196	1,548,120	1,600,888

Robust standard errors in parentheses. All regressions include firm sector, year and HQ commuting area fixed effects. Regressions were estimated using a probit model. Numbers correspond to estimated parameters, not to marginal effects. Wages are measured following the methodology described in Section **??**, but include a municipality-year fixed effect instead of an occupation-establishment-year fixed effect. *** p <0.01, ** p <0.05, * p <0.1.

Column (1) shows that firms are more likely to have an establishment in municipalities that are closer to the firm's HQ. Column (2) shows that this probability also increases if the wages in the municipality are lower than the wages at the HQ's location. The interaction between relative wages and distance from Column (3) indicates that a lower relative wage offsets the impact of distance: A firm might be willing to have an establishment in a location far from its HQ if it offers a big enough cost advantage. However, if a municipality is farther away and is also more expensive, the probability of locating there is even lower. Column (3) also suggests that firms with HQ located in municipalities with high population density tend to have more establishments in general, which could suggest an important role for HQ agglomeration economies. In Columns (4) to (7) we split the sample into four time periods. Even though the intuition from Column (3) abides, note that the parameter accompanying the interaction between the distance and wage gaps is increasing over time. These parameters are significantly different across periods. This trend could suggest that over time the offsetting relation between lower relative wages and distance has become stronger: Firms are more willing to substitute long distances for lower wages now relative to the 1980s.

In Table A9, we estimate similar regressions but use OLS and the number of establishments in each municipality as the dependent variable. The results of these regressions point to the same intuition.

Table A9: Location Probability and Number of Establishments

	Number of Establishments in k of a Firm with HQ in l						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Variables \ Period	1981-2016	1981-2016	1981-2016	1981-1989	1990-1998	1999-2007	2008-2016
Dist to HQ	-0.834***	-0.853***	-0.868***	-0.923***	-0.887***	-0.850***	-0.829***
	(0.039)	(0.037)	(0.038)	(0.038)	(0.038)	(0.041)	(0.038)
$log(W_l/W_k)$		4.025***	-19.159***	-1.254	-20.267***	-24.618***	-24.176***
		(1.480)	(5.645)	(7.294)	(6.279)	(5.321)	(6.914)
$log(W_l/W_k)$*Dist to HQ			4.910***	1.827	5.451***	5.643***	5.710***
			(1.282)	(1.730)	(1.441)	(1.132)	(1.475)
Pop Density at HQ			0.022	0.022	0.027	0.050	0.027
			(0.037)	(0.032)	(0.033)	(0.045)	(0.042)
N		5,946,391		1,355,187	1,442,196	1,548,120	1,600,888

Robust standard errors in parentheses. All regressions include firm sector, year and HQ commuting area fixed effects. Regressions were estimated using OLS. Wages are measured following the methodology described in Section ??, but include a municipality-year fixed effect instead of an occupation-establishment-year fixed effect. *** $p <0.01$, ** $p <0.05$, * $p <0.1$.

To analyze whether our model captures this intuition, we analyze numerically what happens with the number of establishments and their locations when wage differences across locations, communication costs and agglomeration economies change. We also consider the case in which the firm can open a third establishment, keeping fixed the number of locations (o,s). Table A10 in the Appendix shows the value of the parameters used for the simulations and its source. These parameters are either estimated in Section 6.2 or are taken from the data.

1. Communication Costs and Wage Gaps

Consider first what happens with the optimal number of establishments and their locations when communication costs and wage gaps across locations change. Define $g_l \equiv w_o/w_s$ and $g_m \equiv v_o/v_s$, with g_l or g_m larger than one if the wages of workers or managers are higher at the firm's HQ relative to location s. In Panel A of Figure A6, we fix the wage gap of managers g_m and allow τ to vary between 0 and 1 and the wage gap of workers g_l to vary between 0.8 and 1.3. In Panel B, we fix g_l and allow g_m to vary between 0.8 and 1.3. In Panel C, we fix τ and allow g_m and g_l to vary between 0.8 and 1.3. In all simulations, Roman numerals in the legend denote the firm's total number of establishments, with their locations in the subsequent parentheses.

Panels A and B show illustrate important insights. First, notice that for a given level of wage gap (regardless of the type of worker), lower communication costs ($\uparrow \tau$) lead to a higher number of establishments–that is, higher fragmentation. Second, firms are more likely to be fragmented, and are more sensitive to reductions in communication costs, when wages in location o are higher

Table A10: Parameters for Simulations

Parameter	Value	Description	Source
I. Production Function			
α	0.447	Worker share	Table 5
β	0.340	Managerial bundle share	Table 5
λ	0.579	Leontief coef. of local mgr.	Average wage costs of production managers relative to total managerial wage costs.
θ	1.0	Leontief coef. of τH	Normalization
II. Prices			
p	3.6	Price of final good	
v_o	1.075	Managerial wage at o	Exponent of the coefficient of the 1994 HQ dummy from a wage equation (similar to Equation (1)) for managers.
w_o	0.986	Worker wage at o	Exponent of the coefficient of the 1994 HQ dummy from a wage equation (similar to Equation (1)) for workers.
g_m	1.043	Managerial wage gradient	Ratio of v_o and v_s, which is built equivalently but using the 1994 non-HQ dummy
g_l	0.985	Worker wage gradient	Ratio of w_o and w_s, which is built equivalently but using the 1994 non-HQ dummy
f_s	1.0	Fixed cost at s	Normalization
ϕ	0.813	Fixed cost gradient	90th-10th ratio of municipality level hedonic price indices for 1994.
III. Other			
A_i	1.0	Firm Productivity	Normalization
μ_o	1.065	Agglomeration econ. in o	Copenhagen wage premium, computed from a wage equation similar to Equation (1), but with worker fixed effects.
μ_s	1.0	Agglomeration econ. in s	Normalization

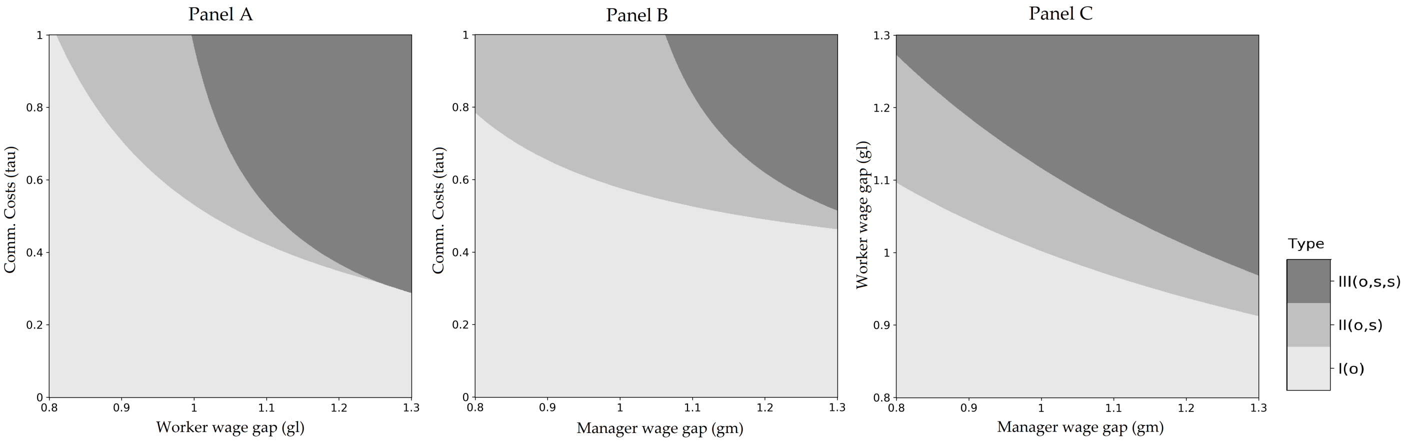
This table shows the parameters used in Figures A6 to A10, their descriptions and source. More detailed information is available on request

than wages in s. Finally, Panel C shows that when workers and managers are relatively cheap at the HQ location, the firm will choose to be in a centralized equilibrium. However, as location s becomes cheaper, the firm will start opening more establishments outside the central location. This effect is stronger if wage differences increase for both types of workers. Finally, note that under our current set of parameters, if the wage of production workers is very low in s relative to o, the firm will open establishments in s, almost regardless of the managerial wage gap.

2. Communication Costs and Agglomeration Economies

Consider now what happens with the optimal number of establishments when communication costs and managerial agglomeration economies change. In particular, we plot the number of establishments chosen by the firm for different values of $\tau \in [0, 1]$ and $\mu \in [0.75, 2]$. Panel A in Figure A7 shows the scenario in which the firm chooses between being a centralized firm (with HQ in o) or opening a second establishment. In this particular case, for lower levels of agglomeration economies in location $j = o$, cheaper communication costs would lead the firm to move from a centralized to a fragmented equilibrium. For higher levels of agglomeration economies, cheaper communication costs can cause the movement of the non-HQ establishment

Figure A6: Communication Costs and Wage Gaps



These figures show simulations of the firm's optimal number of establishments and their locations, when wages across locations and communication cost change. The simulations are based on equations (23) and (28). Roman numerals in the legend denote the firm's total number of establishments, with their locations in the subsequent parentheses.

from the central to the satellite location. In either case, a decrease in communication costs leads to spatial decentralization.

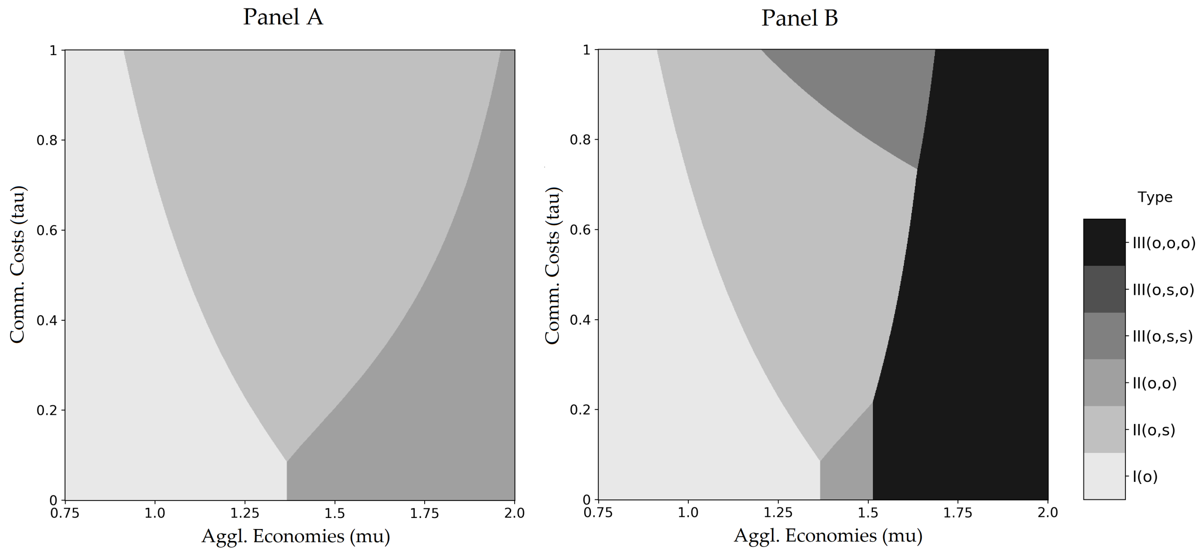
On the other hand, and perhaps quite counterintuitive, is the relation between the number of establishments and the agglomeration parameter μ. Note that for some levels of communication costs, higher μ leads to the creation of a second establishment in s. This is because when managerial agglomeration economies increase, the firm would like to hire more managers and workers to produce more final good and HQ services. However, given the existence of diminishing marginal returns, the marginal benefit of this increase in the total number of employees might not be larger than the marginal cost, unless the firm opens a second establishment. Thus, the firm might opt for the fragmented equilibrium if communication costs are low enough. As agglomeration becomes even stronger, the firm would like to move the second establishment back to the central location.

In Panel B we analyze the case in which the firm can open a third establishment in either of the two locations. In this case, for a given value of communication costs, higher agglomeration economies lead the firm to move from having one, to having two, to having three establishments. A similar intuition applies if communication costs decrease, for a wide range of levels of agglomeration economies. Finally, if agglomeration economies are particularly strong, the firm always chooses to have three establishments regardless of the level of communication costs, with most (if not all) in the central location.

In Figures A8 and A9, we show that the average number of establishments per firm and the average distance to the HQ increases as communication costs decrease. These figures resemble the empirical results from Acosta and Lyngemark (2021).

Finally, we explore what happens to the optimal number of establishments when firm productivity and communication cost change. Panels A and B from Figure A10 show that at high levels of communication costs, only highly productive firms open additional establishments, and these establishments are in the same location as their HQ. Furthermore, as communication costs decrease, a group of firms in the middle of the productivity distribution also decide to open additional establishments, mostly in s, where both land and labor are cheaper.

Figure A7: Communication Costs and Agglomeration Economies



These figures show simulations of the firm's optimal number of establishments and their locations, when communication cost and agglomeration economies change. The simulations are based on equations (23) and (28). Roman numerals in the legend denote the firm's total number of establishments, with their locations in the subsequent parentheses.

Figure A8: Average Number of Establishments and Communication Costs

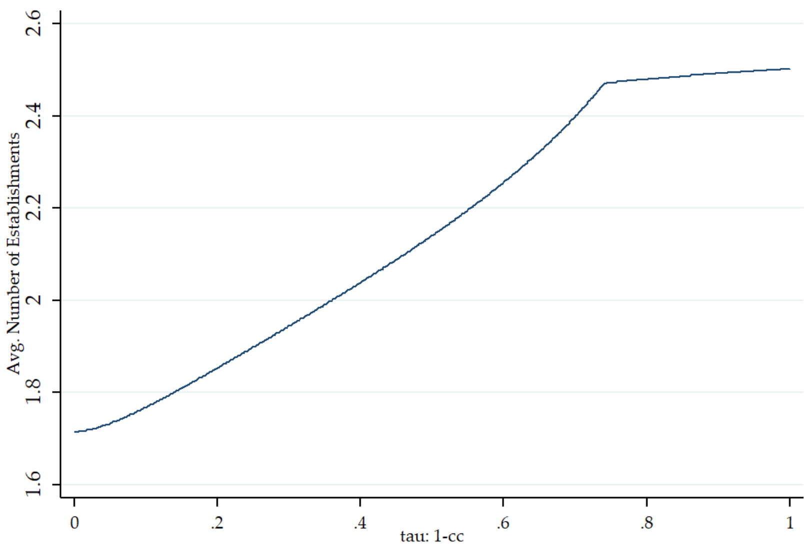


Figure A9: Distance to HQ and Communication Costs

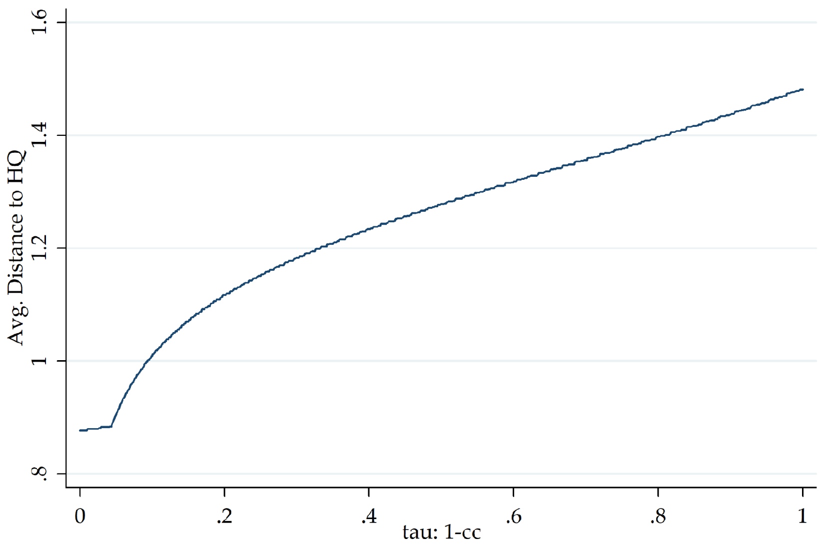
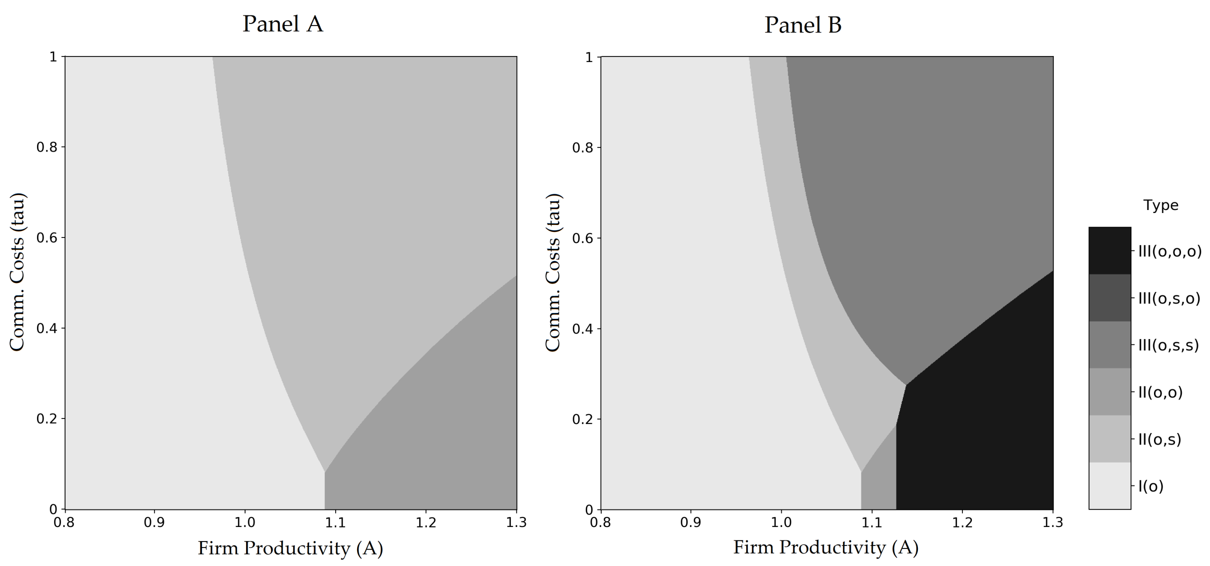


Figure A10: Firm Productivity and Communication Costs



These figures show simulations of the firm's optimal number of establishments and their locations, when communication cost and firm productivity change. The simulations are based on equations (23) and (28). Roman numerals in the legend denote the firm's total number of establishments with their locations in the subsequent parentheses.